High-Dimensional Causal Inference

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High-Dimensional Causal Inference

1. Treatment effect heterogeneity
   - How do treatment effects vary across individuals?
   - Who benefits from (or is harmed by) the treatment?
2. Treatment heterogeneity
   - What aspects of a treatment are responsible for causal effects?
   - What combination of treatments is efficacious?
3. Individualized treatment regimes
   - What combination of treatments is optimal for a given individual?

- Causal prediction vs. causal learning in high-dimension
- Importance of interpretability in high-dimension

Factorial Designs with Many Treatments

- The most basic form of high-dimensional causal inference
- Many treatments, each having multiple levels
- A motivating application: Conjoint analysis (Hainmueller *et al.* 2014)
  - survey experiments to measure immigration preferences
  - a representative sample of 1,396 American adults
  - each respondent evaluates 5 pairs of immigrant profiles
  - $\texttt{gender}^2$, $\texttt{education}^7$, $\texttt{origin}^{10}$, $\texttt{experience}^4$, $\texttt{plan}^4$, $\texttt{language}^4$, $\texttt{profession}^{11}$, $\texttt{application reason}^3$, $\texttt{prior trips}^5$
  - Over 1 **million** treatment combinations!
  - What combinations of immigrant characteristics make them preferred?
- Too many treatment combinations $\rightsquigarrow$ Need for an effective summary
- Many potential applications in academia and industry

Machine Learning and Causal Inference

- How should we analyze the data from a factorial randomized experiment with many treatments?
- Regression model: $\mathbb{E}(Y \mid \mathbf{T}) = f(\mathbf{T})$
- There are many machine learning methods to estimate this model
- In this setting, causal inference is a prediction problem $\mathbb{E}(Y(\mathbf{t})) = \mathbb{E}(Y \mid \mathbf{T} = \mathbf{t}) = f(\mathbf{t})$
- But, how do we interpret these models?
- Scientists wish to understand the causal structure
  1. Predict $\mathbb{E}(Y(\mathbf{t}))$ using each treatment combination $\mathbf{t}$ and look at what values of $\mathbf{t}$ yield high/low predicted values of Y
     $\rightsquigarrow$ Finding patterns is difficult in high dimension
  2. Use a sparse regression model
     $\rightsquigarrow$ Difficult to interpret interaction terms (lack of invariance to the baseline condition)
- Causal testing vs. causal exploration

Causal Effects with Two Multi-valued Treatment Variables

1. Average Treatment Combination Effect (ATCE):
   - Average effect of treatment combination $(A, B) = (a_\ell, b_m)$ relative to the baseline condition $(A, B) = (a_0, b_0)$

   $$\tau(a_\ell, b_m; a_0, b_0) \equiv \mathbb{E}\{Y(a_\ell, b_m) - Y(a_0, b_0)\}$$

   - Which treatment combination is most efficacious?

2. Average Marginal Treatment Effect (AMTE; Hainmueller et al. 2014):
   - Average effect of treatment $A = a_\ell$ relative to the baseline condition $A = a_0$ averaging over the other treatment B

   $$\psi(a_\ell, a_0) \equiv \int_{\mathcal{B}} \mathbb{E}\{Y(a_\ell, B) - Y(a_0, B)\} dF(B)$$

   - Which treatment is effective on average?

Other treatments can be integrated out

3. Average Marginal Treatment Interaction Effect (**AMTIE**):

$$\pi(a_\ell, b_m; a_0, b_0) \equiv \underbrace{\tau(a_\ell, b_m; a_0, b_0)}_{\text{ATCE of } (A,B)=(a_\ell, b_m)} - \underbrace{\psi(a_\ell, a_0)}_{\text{AMTE of } a_\ell} - \underbrace{\psi(b_m, b_0)}_{\text{AMTE of } b_m}$$

- Additional effect induced by $A = a_\ell$ and $B = b_m$ together beyond the separate effect of $A = a_\ell$ and that of $B = b_m$
- Unlike the standard interaction effects, the **AMTIE**s are invariant to the choice of baseline category
- However, the **AMTE**s and **AMTIE**s do depend on the distribution of treatment assignment
- Two solutions:
  1. use the treatment assignment probabilities from the experiment
  2. use the distribution of treatments in the target population

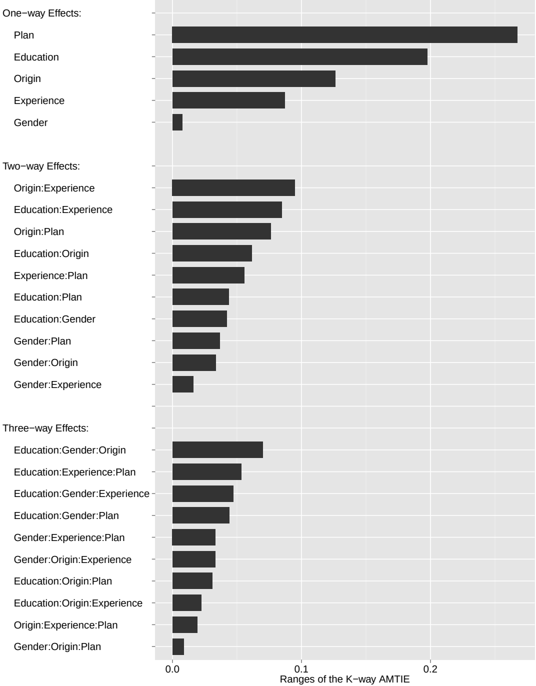
Causal Interaction in High-Dimension

- Definition: the difference between the ATCE and the sum of lower-order **AMTIE**s

- Example: 3-way **AMTIE**, $\pi_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})$, equals

$$\underbrace{\tau_{1:3}(t_1, t_2, t_3; t_{01}, t_{02}, t_{03})}_{\text{ATCE}}$$
$$- \underbrace{\left\{\pi_{1:2}(t_1, t_2; t_{01}, t_{02}) + \pi_{2:3}(t_2, t_3; t_{02}, t_{03}) + \pi_{1:3}(t_1, t_3; t_{01}, t_{03})\right\}}_{\text{sum of 2-way AMTIEs}}$$
$$- \underbrace{\left\{\psi(t_1; t_{01}) + \psi(t_2; t_{02}) + \psi(t_3; t_{03})\right\}}_{\text{sum of (1-way) AMTEs}}$$

- Contrast this with the standard higher-order interaction:
  3-way interaction effect = difference between 2-way interaction effects



- Sparse regression with one-way, two-way, and three-way effects
- Range of **AMTIE**s: importance of each factor and factor interaction
- Sparcity-of-effects principle
- `gender` appears to play a significant role in three-way interactions

3-way Interaction (education $\times$ gender $\times$ origin)

$$\underbrace{\tau(\texttt{Graduate}, \texttt{Male}, \texttt{India}; \texttt{Graduate}, \texttt{Female}, \texttt{India})}_{7.46} \qquad (n = 52; \quad n = 40)$$

$$= \underbrace{\psi(\texttt{Male}; \texttt{Female})}_{-0.77} + \underbrace{\pi(\texttt{Graduate}, \texttt{Male}; \texttt{Graduate}, \texttt{Female})}_{-0.34}$$

$$+ \underbrace{\pi(\texttt{Male}, \texttt{India}; \texttt{Female}, \texttt{India})}_{1.56} + \underbrace{\pi(\texttt{Graduate}, \texttt{Male}, \texttt{India}; \texttt{Graduate}, \texttt{Female}, \texttt{India})}_{7.01}$$

$$\underbrace{\tau(\texttt{High school, Male, Germany; High school, Female, Germany})}_{-11.52}$$

$(n = 41; \quad n = 56)$

$$= \underbrace{\psi(\texttt{Male}; \texttt{Female})}_{-0.77} + \underbrace{\pi(\texttt{High school, Male; High school, Female})}_{-0.67}$$

$$+ \underbrace{\pi(\texttt{Male}, \texttt{Germany}; \texttt{Female}, \texttt{Germany})}_{-3.34}$$

$$+ \underbrace{\pi(\texttt{High school, Male,Germany; High school, Female,Germany})}_{-6.74}.$$

Challenges of High-dimensional Causal Inference

- Estimation and inference $\rightsquigarrow$ machine learning and statistics
- Interpretation $\rightsquigarrow$ causal inference
- Experimental design
  - Multi-armed bandits in high-dimension
  - More noise $\rightsquigarrow$ sensitivity to the choice of tuning parameter
  - Linear UCB with variable selection $\rightsquigarrow$ attains oracle properties
  - Issues of dynamic variable selection in high-dimension