Spatiotemporal causal inference with arbitrary spillover and carryover effects

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Motivation

- Increasing availability of unstructured data in social sciences
  - don't come in a nice matrix form $\leftrightsquigarrow$ survey, official statistics
  - text, images, audio, video, etc.
- How should we draw causal inference from these new types of data?
- Causal inference with spatio-temporal data
  - a time series of maps as data
  - treatment and outcome event locations in a continuous space
  - applications: crime, disease, disasters, pollution, etc.
- Methodological challenges
  1. spillover effects over space
  2. carryover effects over time
  3. infinitely many possible treatment and outcome locations
- Current practice
  1. arbitrary discretization of space
  2. strong assumptions about spillover and carryover effects

Contributions

- Causal inference with spatio-temporal data
  - impossible to estimate causal effects of each treatment event
    - unrestricted spillover and carryover effects
    - probability of each treatment realization is zero $\rightsquigarrow$ lack of overlap
  - stochastic intervention based on the distribution of treatments
- Causal estimands under stochastic intervention
  - expected number of outcome events within a region of interest
  - various stochastic interventions
    1. change the dosage while keeping the distribution identical
    2. change the distribution while keeping the dosage constant
    3. intervention over multiple time periods
- The proposed methodology can estimate:
  - average treatment effects (this talk)
  - heterogeneous treatment effects (in the paper)
  - causal mediation effects (in the paper)
- Empirical application: airstrikes and insurgent violence in Iraq

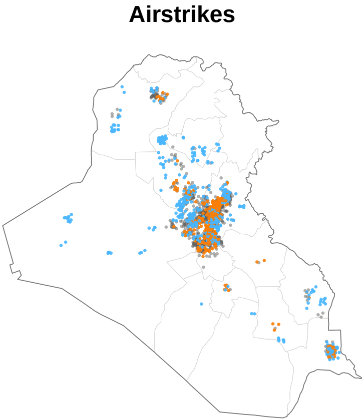
Impacts of Airstrikes on Insurgent Violence in Iraq

- Airstrikes as a principal tool for combating insurgency in civil wars

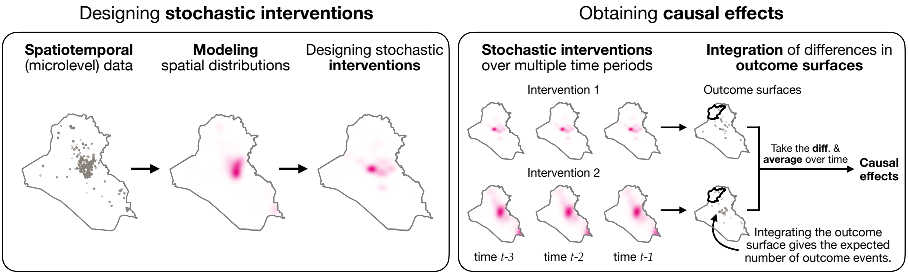
- Three ongoing debates:
  1. overall effectiveness: do airstrikes reduce subsequent insurgent attacks?
  2. heterogeneous effects: what factors moderate effects of airstrikes?
  3. causal mechanisms: does civilian casualty mediate effects of airstrikes?

- American air campaign in Iraq:
  - declassified USAF data from Feb. 2007 to July 2008 ("surge" period)
  - date and precise geolocation for
    - airstrikes: aircraft type, number and type of bombs
    - insurgent attacks: small arms fire, improvised explosive devices
  - location of US and UK military units
    - weekly, district-level
    - troop density: soldiers per 1,000 residents
    - troop type: US Marines, US Army, and UK Army

Data: Airstrikes and Insurgent Attacks



Methodological Overview



1. Model treatment assignment mechanism
2. Design stochastic interventions of interest
3. Estimate the counterfactual outcomes and average them over time

The Setup

- T time periods: $t = 1, 2, \ldots, T$
- Treatment variable
  - Ω: set of all possibly infinite locations that can receive the treatment
  - $W_t(s) \in \{0, 1\}$: binary treatment indicator for location s at time t
  - $W_t = \{W_t(s) : s \in \Omega\} \in \mathcal{W}$: treatment location map at time t
  - $S_{W_t} = \{s \in \Omega : W_t(s) = 1\}$: set of treatment-active locations at time t
  - $\overline{\boldsymbol{W}}_t = (W_1, W_2, \ldots, W_t)$: observed treatment history up to time t
- Outcome variable
  - $Y_t(s)$, Y_t, and $\overline{\boldsymbol{Y}}_t$ can be similarly defined
  - Potential outcome: $Y_t(\overline{\boldsymbol{w}}_t)$ where $w_t \in \mathcal{W}$ is a realized treatment and $\overline{\boldsymbol{w}}_t = (w_1, w_2, \ldots, w_t) \in \mathcal{W}^t$ is a treatment history realization at time t
  - Observed outcome: $Y_t = Y_t(\overline{\boldsymbol{W}}_t)$
  - $S_{Y_t(\overline{\boldsymbol{w}}_t)}$: set of outcome-active locations under treatment history $\overline{\boldsymbol{w}}_t$
  - History of all potential outcomes up to time t: $\overline{\mathcal{Y}}_t = \{Y_{t'}(\overline{\boldsymbol{w}}_{t'}) : \overline{\boldsymbol{w}}_{t'} \in \mathcal{W}^{t'}, t' \leq t\}$
- Time-varying confounders: X_t, $\overline{\boldsymbol{X}}_t$, $X_t(\overline{\boldsymbol{w}}_{t-1})$, and $\overline{\mathcal{X}}_t$

Causal Estimands

- Stochastic intervention: any distribution of treatment can be used
- We consider Poisson point process F_h with intensity function h
- Expected number of outcome-active locations in region B at time t under stochastic intervention F_h conducted at time t

$$\overline{N}_{Bt}(F_h) \; = \; \int_{\mathcal{W}} N_B(Y_t(\overline{\boldsymbol{W}}_{t-1}, w_t)) dF_h(w_t)$$

- Further average this quantity over time:

$$\overline{N}_B(F_h) \; = \; \frac{1}{T} \sum_{t=1}^{T} \overline{N}_{Bt}(F_h)$$

- We can compare the different interventions:

$$\tau_B(F_{h'}, F_h) \; = \; \overline{N}_B(F_{h'}) - \overline{N}_B(F_h)$$

Stochastic Intervention over Multiple Time Periods

- Consider a non-dynamic stochastic intervention over L time periods

$$F_{\boldsymbol{h}} \;=\; F_{h_1} \times \cdots \times F_{h_L} \quad \text{where } \boldsymbol{h} = (h_1, h_2, \ldots, h_L)$$

- Expected number of outcome-active locations in region B at time t under stochastic intervention $F_{\boldsymbol{h}}$ conducted from time $t - L + 1$ to t

$$\overline{N}_{Bt}(F_{\boldsymbol{h}}) \;=\; \int_{\mathcal{W}} \cdots \int_{\mathcal{W}} N_B(Y_t(\overline{\boldsymbol{W}}_{t-L}, w_{t-L+1}, \ldots, w_t)) \\ dF_{h_L}(w_{t-L+1}) \cdots dF_{h_1}(w_t)$$

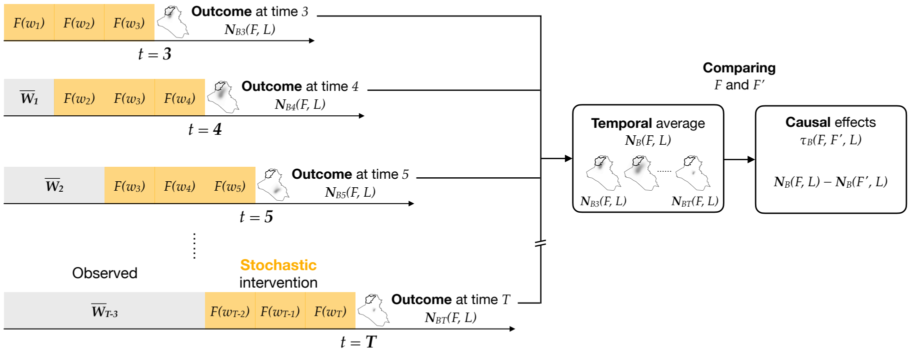
- Average this quantity over time:

$$\overline{N}_B(F_{\boldsymbol{h}}) \;=\; \frac{1}{T - L + 1} \sum_{t=L}^{T} \overline{N}_{Bt}(F_{\boldsymbol{h}})$$

- Comparison of different interventions:

$$\tau_B(F_{\boldsymbol{h}'}, F_{\boldsymbol{h}}) \;=\; \overline{N}_B(F_{\boldsymbol{h}'}) - \overline{N}_B(F_{\boldsymbol{h}})$$

Recap



- Each counterfactual outcome is conditional on the past
- Averaging is done over time
- Inference is done by letting T go infinity
- Example of causal inference with time series

Assumptions

1. Unconfoundedness: treatment is independent of all potential (past and future) paths for the outcome and time-varying confounders conditional on the observed history

$$f(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t, \{\overline{\mathcal{Y}}_T, \overline{\mathcal{X}}_T\}) \;=\; f(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)$$

2. Overlap: there exists a constant $\delta_W > 0$ such that

$$\underbrace{f(W_t = w \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)}_{propensity\;\;score} \;>\; \delta_W \cdot \underbrace{f_h(w)}_{density\;\;of\;\;F_h} \qquad \text{for all } w \in \mathcal{W}$$

$\rightsquigarrow$ the ratio $f_h(w)/f(W_t = w \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)$ is bounded

The Proposed Estimator

- Inverse probability of treatment weighting (IPW)
- Kernel smoothing of spatial point patterns
- Estimated outcome surface at $\omega \in \Omega$ under the intervention F_h

$$\widehat{Y}_t(F_h;\omega) \; = \; \underbrace{\frac{\overbrace{f_h(W_t)}^{counterfactual\;\; distribution}}{\hat{f}(W_t \mid \overline{\boldsymbol{W}}_{t-1}, \overline{\boldsymbol{Y}}_{t-1}, \overline{\boldsymbol{X}}_t)}}_{actual\;\; distribution} \; \underbrace{\sum_{s \in S_{Y_t}} K_b(\|\omega - s\|)}_{spatially\;\; smoothed\;\; outcome}$$

  where K_b is the scaled Kernel function with bandwidth parameter b
- Estimated number of outcome-active locations in region B

$$\widehat{\widehat{N}}_{Bt}(F_h) \; = \; \int_B \widehat{Y}_t(F_h;\omega) d\omega$$

- Averaging over time

$$\widehat{\widehat{N}}_B(F_h) \; = \; \frac{1}{T} \sum_{t=1}^{T} \widehat{\widehat{N}}_{Bt}(F_h)$$

Estimation for Intervention over Multiple Time Periods L

- Estimated outcome surface at $\omega \in \Omega$

$$\widehat{Y}_t(F_{\boldsymbol{h}};\omega) \;=\; \underbrace{\prod_{j=t-L+1}^{t} \frac{f_{h_{t-j+1}}(W_j)}{\hat{f}(W_j \mid \overline{\boldsymbol{W}}_{j-1}, \overline{\boldsymbol{Y}}_{j-1}, \overline{\boldsymbol{X}}_j)}}_{product\ \ of\ \ L\ \ ratios} \sum_{s \in S_{Y_t}} K_b(\|\omega - s\|)$$

- Estimated number of outcome-active locations in region B

$$\widehat{\widehat{N}}_{Bt}(F_{\boldsymbol{h}}) \;=\; \int_B \widehat{Y}_t(F_{\boldsymbol{h}};\omega)d\omega$$

- Averaging over time

$$\widehat{\widehat{N}}_B(F_{\boldsymbol{h}}) \;=\; \frac{1}{T-L+1}\sum_{t=L}^{T} \widehat{\widehat{N}}_{Bt}(F_{\boldsymbol{h}})$$

- Asymptotic normality

$$\sqrt{T}\left(\widehat{\widehat{N}}_B(F_{\boldsymbol{h}}) - \overline{N}_B(F_{\boldsymbol{h}})\right) \xrightarrow{d} \mathcal{N}(0, v)$$

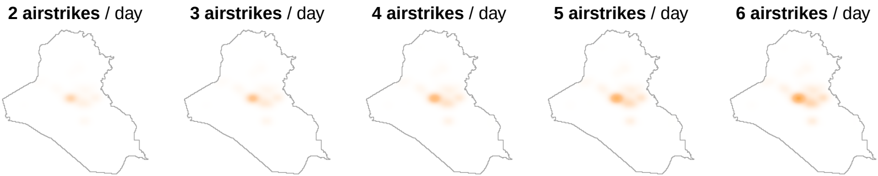
- Hájek estimator (normalized weights) for efficiency

Empirical Analysis: Setup

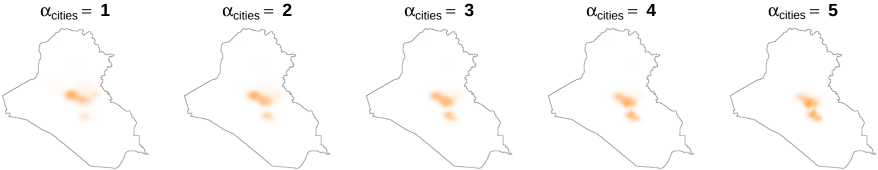
- Estimate the baseline treatment distribution f_0
  - inhomogeneous Poisson process regression
  - 2006 data, separate from the 2007 evaluation data
  - covariates: aid, histories of air strikes, show of force, and insurgent attacks (1, 7, and 30 days), log population, time splines, distances from rivers, major roads, cities, and settlements

- Questions:
  1. How does increasing airstrikes affect insurgent violence?
     $\rightsquigarrow$ vary $c > 0$ for $h(\omega) = c \cdot f_0(\omega)$
  2. How does the shift in the prioritization of certain locations for airstrikes change the spatial pattern of insurgent attacks?
     $\rightsquigarrow$ vary $\alpha > 0$ for $h_\alpha(\omega) \propto f_0(\omega) d_\alpha(\omega)$ with $\int_\Omega h_\alpha(\omega) d\omega = c$
     - power density $d_\alpha(\omega) \propto d(\omega)^\alpha$
     - $d(\omega) =$ the normal density centered at s_f with precision α

Intervention by Picture

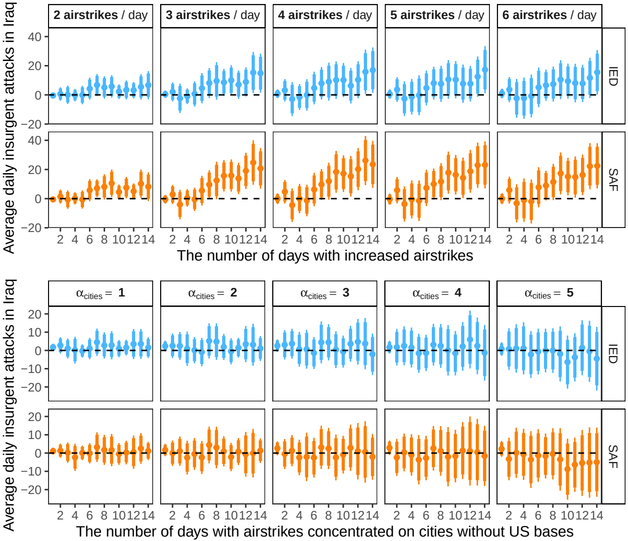


(a) Counterfactual interventions with intensified airstrikes



(b) Counterfactual interventions with location shifts

Increasing the Expected Number of Airstrikes from 1 to 6 per Day Leads to More Insurgent Attacks with Large *L*



Concluding Remarks

- A new approach to causal inference with spatio-temporal data
  - directly model point patterns without arbitrary aggregation
  - allow for unstructured spillover and carryover effects
- Key idea: stochastic intervention
  - consider treatment distributions rather than fixed treatment values
  - can handle infinitely many possible treatment locations
- Three methods
  1. average treatment effects
  2. heterogeneous treatment effects
  3. causal mediation effects
- R package: geocausal available at CRAN
- Paper at https://imai.fas.harvard.edu/research/spatiotempo.html