Discussion of "Estimation and Inference in Boundary Discontinuity Designs" by Cattaneo, Titiunik, and Yu

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Contributions

- Wide applicability of boundary RDD
  - geographical boundaries of administrative units
  - program eligibility criteria based on two continuous scores
- Heterogeneous treatment effect estimation under boundary RDD
  - Bias of distance-based estimator due to non-smooth boundary
  - Point-wise and uniform inference under boundary RDD
- Software development to assist applied researchers
- Another awesome paper from the best RDD team in the world
- A couple of questions for the authors

What about the average treatment effect?

- Definition:

$$\text{ATE} \;:= \mathbb{E}[Y(1) - Y(0) \mid \boldsymbol{X} \in \mathcal{B}]$$

- Bivariate location-based estimation:

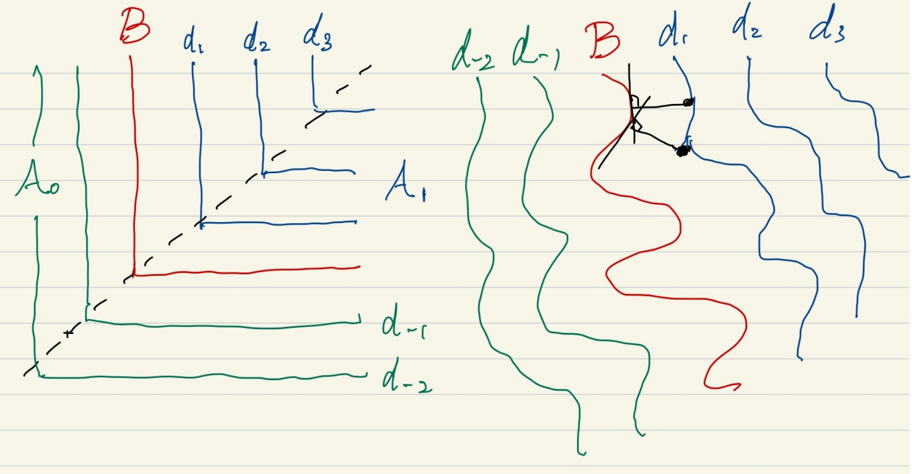
$$\text{ATE} \;=\; \frac{\int_{\mathcal{B}} \mathbb{E}[Y(1) - Y(0) \mid \boldsymbol{X} = \boldsymbol{x}] f(\boldsymbol{x}) d\boldsymbol{x}}{\int_{\mathcal{B}} f(\boldsymbol{x}) d\boldsymbol{x}}$$

density estimation along the boundary may be difficult due to sparcity

- Univariate distance-based estimation:

$$\text{signed distance function:} \quad D(\boldsymbol{x}) = \begin{cases} d(\boldsymbol{x}, \mathcal{B}) & \text{if } \boldsymbol{x} \in \mathcal{A}_1 \\ -d(\boldsymbol{x}, \mathcal{B}) & \text{if } \boldsymbol{x} \notin \mathcal{A}_1 \end{cases}$$

where $d(\boldsymbol{x}, \mathcal{B}) = \inf_{\boldsymbol{y} \in \mathcal{B}} d(\boldsymbol{x}, \boldsymbol{y})$



- If $\mathcal{B}$ is piecewise smooth, $D(\boldsymbol{x})$ is differentiable almost everywhere
- Fast numerical algorithms exist to compute signed distance function
- Can we use local polynomial regression for $D(\boldsymbol{x})$ to estimate the ATE?

What about spatial RDD?

1. Treatment bundle
   - confounding bias
   - interpretation issue

2. Interference between units

$$\mathbb{E}[Y_i(\mathcal{T}_{\mathcal{B}} = 1_{\mathrm{B}}, \mathcal{T}_{\mathcal{A}_1} = 1_{\mathcal{A}_1}, \mathcal{T}_{\mathcal{A}_0} = 0_{\mathcal{A}_0}) \\ - Y_i(\mathcal{T}_{\mathcal{B}} = 0_{\mathrm{B}}, \mathcal{T}_{\mathcal{A}_1} = 1_{\mathcal{A}_1}, \mathcal{T}_{\mathcal{A}_0} = 0_{\mathcal{A}_0}) \mid \boldsymbol{X} \in \mathcal{B}]$$

   - $\mathcal{T}$: treatment assignments of all (possibly infinite number of) units
   - $\mathcal{T}_{\mathcal{B}}$: treatment assignments of all units on boundary
   - $\mathcal{T}_{\mathcal{A}_a}$: treatment assignments of all units in $\mathcal{A}_a$ for $a = 0, 1$

3. Spatial dependency
   - observations are no longer i.i.d.
   - how to incorporate spatial dependency into uncertainty quantification?