Statistical Analysis of Causal Mechanisms for Randomized Experiments

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- This talk is based on the following paper:

  Imai, Kosuke, Luke Keele, and Teppei Yamamoto.
  "*Identification and Inference in Causal Mediation Analysis*"
  available at http://imai.princeton.edu/

- Help from Dustin Tingley is acknowledged

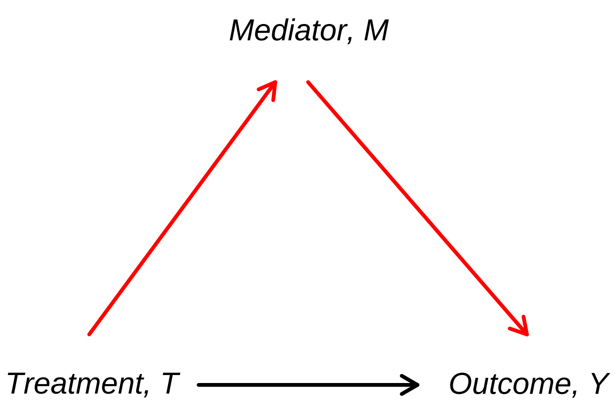
Randomized Experiments and Causal Mechanisms

- Causal inference is a central goal of social science
- Randomized experiments as **gold standard**
- But, experiments are a **black box**
- Can only tell *whether* the treatment causally affects the outcome
- Not *how* and *why* the treatment affects the outcome

- Challenge is how to identify causal mechanisms

What This Talk is About

- **Goal:** Show how to identify causal mechanisms using statistics
- **Method:** Causal Mediation Analysis



- Direct and indirect effects; intermediate and intervening variables
- Popular among social psychologists (e.g., Baron and Kenny)

Current Practice

- Regression

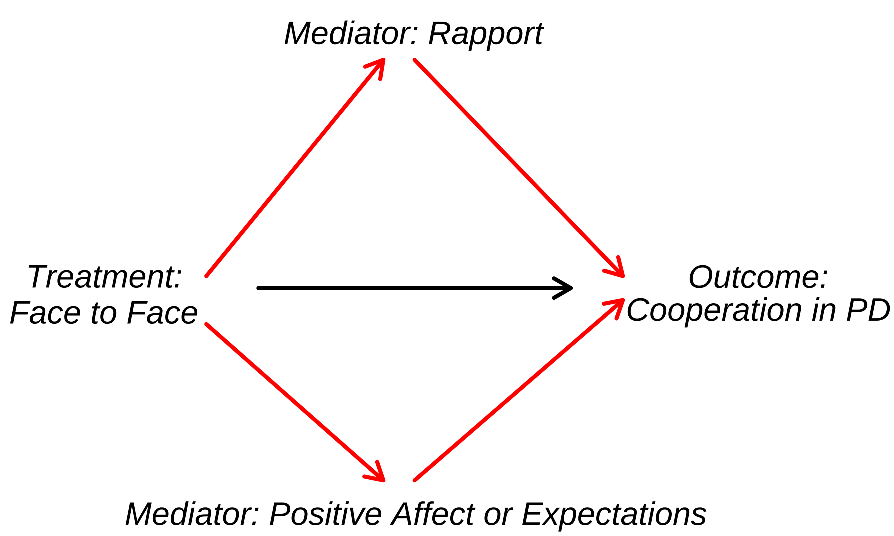
$$Y_i = \alpha + \beta T_i + \gamma M_i + \delta X_i + \epsilon_i$$

- Each coefficient is interpreted as a causal effect
- Sometimes, it's called marginal (or partial) effect
- Idea: increase T_i by one unit while holding M_i and X_i constant

- Post-treatment bias: if you change T_i, that may also change M_i
- Usual advice: only include causally prior (or pre-treatment) variables
- But, then you lose causal mechanisms!

Causal Mediation in Interactive Experiment I

- Communication can influence behavior in strategic games
- But what psychological mechanisms are at work?
- Drolet and Morris (2000). *J. of Experimental Social Psychology*
- Rapport vs. positive affect and expectations

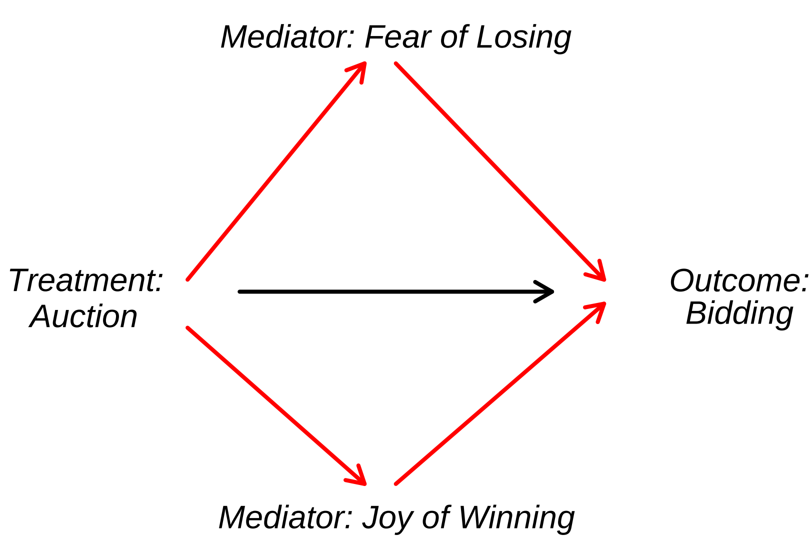


Experimental Design and Finding

- How does rapport differ from positive affect and expectations?
  - shared sense of mutual understanding
  - dyadic or group level process
  - convergence of nonverbal expressions
  - observable by a third party
- Experimental Design:
  - randomized treatment: face-to-face or telephone conversation
  - talk about "positive experiences at Stanford"
  - fill out surveys measuring rapport and positive expectations
  - measure outside observers' perception of rapport
  - play a single shot PD game
- Finding: rapport mediates the positive effects of face-to-face communication, but positive affect and expectations do not

Causal Mediation Analysis in Interactive Experiment II

- People overbit in auctions. Why?
- Useful for designing better auctions
- Delgado *et al.* (2008) *Science*
- Fear of losing vs. Joy of winning



Experimental Design and Findings

- Randomized treatment: lottery or two-person auction
- fMRI to measure BOLD response to outcomes in the right striatum
- Evaluate causal mechanisms of overbidding

- Greater change in BOLD signal when subject lost in auction
- Little change when subject won
- Important mediating role of fear of losing in auction

Formal Statistical Framework of Causal Inference

- Units: $i = 1, \ldots, n$
- "Treatment": $T_i = 1$ if treated, $T_i = 0$ otherwise
- *Observed* outcome: Y_i
- Pre-treatment covariates: X_i
- Potential outcomes: $Y_i(1)$ and $Y_i(0)$ where $Y_i = Y_i(T_i)$

Subject pair i	Communication type T_i	Cooperation $Y_i(1)$	Cooperation $Y_i(0)$	Average age X_{1i}	How many economists X_{2i}
1	1	1	?	20	1
2	0	?	0	21.5	0
3	0	?	1	19	2
⋮	⋮	⋮	⋮	⋮	⋮
n	1	0	?	22	2

- Causal effect: $Y_i(1) - Y_i(0)$

Identification of Causal Effects in Standard Settings

- Nonparametric identification: What can we learn from the data generating process alone?
- Average Treatment Effect (ATE):

$$\tau \equiv \mathbb{E}(Y_i(1) - Y_i(0))$$

- Ignorability (randomization, no omitted variable):

$$(Y_i(1), Y_i(0)) \perp\!\!\!\perp T_i \mid X_i$$

- Identification under ignorability:

$$\tau \;=\; \mathbb{E}(Y_i \mid T_i = 1, X_i) - \mathbb{E}(Y_i \mid T_i = 0, X_i)$$

- Relationship with the linear regression:

$$Y_i(T_i) = \alpha + \beta T_i + \gamma X_i + \epsilon_i$$

  where ignorability implies $T_i \perp\!\!\!\perp \epsilon_i \mid X_i$

Notation for Causal Mediation Analysis

- Binary treatment: $T_i \in \{0, 1\}$
- Mediator: M_i
- Outcome: Y_i
- Observed covariates: X_i

- Potential mediators: $M_i(t)$ where $M_i = M_i(T_i)$
- Potential outcomes: $Y_i(t, m)$ where $Y_i = Y_i(T_i, M_i(T_i))$

Defining and Interpreting Causal Mediation Effects

- Total causal effect: $\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0))$
- Causal mediation effects:

$$\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Change the mediator from $M_i(0)$ to $M_i(1)$ while holding the treatment constant at t
- Indirect effect of the treatment on the outcome through the mediator under treatment status t
- $Y_i(t, M_i(t))$ is observable but $Y_i(t, M_i(1-t))$ is not

- Direct effects:

$$\zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))$$

- Change the treatment from 0 to 1 while holding the mediator constant at $M_i(t)$
- Total effect = mediation (indirect) effect + direct effect:

$$\tau_i = \delta_i(t) + \zeta_i(1-t)$$

- Quantities of interest: Average Causal Mediation Effects,

$$\bar{\delta}(t) \equiv \mathbb{E}(\delta_i(t)) = \mathbb{E}\{Y_i(t, M_i(1)) - Y_i(t, M_i(0))\}$$

The Main Identification Result

Assumption 1 (Sequential Ignorability)

$$\{Y_i(t,m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i,$$
$$Y_i(t,m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

for $t = 0, 1$ and $m \in \mathcal{M}$.

Theorem 1 (Nonparametric Identification)

Under Assumption 1, for $t = 0, 1$,

$$\bar{\delta}(t) = (-1)^t \int \left\{ \int \mathbb{E}(Y_i \mid M_i, T_i = t, X_i)\, dP(M_i \mid T_i = 1 - t, X_i) - \mathbb{E}(Y_i \mid T_i = t, X_i) \right\} dP(X_i)$$

Theoretical and Practical Implications

- Existing statistics literature concludes that an additional assumption is required for the identification of mediation effects
- However, sequential ignorability *alone* is sufficient
- Fit two nonparametric regressions:
  1. $\mu_{tm}(x) \equiv \mathbb{E}(Y_i \mid T_i = t, M_i = m, X_i = x)$
  2. $\lambda_{tm}(x) \equiv \Pr(M_i = m \mid T_i = t, X_i = x)$
- The plug-in estimator for a discrete mediator:

$$(-1)^t \left\{ \sum_{m=0}^{J-1} \frac{\sum_{i=1}^n \mathbf{1}\{T_i = 1-t\} \hat{\lambda}_{1-t,m}(X_i) \sum_{i=1}^n \mathbf{1}\{T_i = t\} \hat{\mu}_{tm}(X_i) \hat{\lambda}_{tm}(X_i)}{n_{1-t} \sum_{i=1}^n \mathbf{1}\{T_i = t\} \hat{\lambda}_{tm}(X_i)} - \frac{1}{n_t} \sum_{i=1}^n \mathbf{1}\{T_i = t\} \left(\sum_{m=0}^{J-1} \hat{\mu}_{tm}(X_i) \hat{\lambda}_{tm}(X_i) \right) \right\}.$$

Identification under Linear Structural Equation Model

Theorem 2 (Identification under LSEM)

Consider the following linear structural equation model

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i}. \end{aligned}$$

Under Assumption 1, the average causal mediation effects are identified as $\bar{\delta}(0) = \bar{\delta}(1) = \beta_2\gamma$.

- Run two (not three) regressions and multiply two coefficients!
- Direct effect: β_3
- Total effect: $\beta_2\gamma + \beta_3$
- Total effect could be zero even when mediation effects are not

Need for Sensitivity Analysis

- The sequential ignorability assumption is often too strong
- Need to assess the robustness of findings via sensitivity analysis
- Question: How large a departure from the key assumption must occur for the conclusions to no longer hold?
- Parametric and nonparametric sensitivity analysis by assuming

$$\{Y_i(t,m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i$$

but not

$$Y_i(t,m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

Parametric Sensitivity Analysis

- Sensitivity parameter: $\rho \;\equiv\; \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i})$
- Existence of omitted variables leads to non-zero ρ
- Set ρ to different values and see how mediation effects change
- All you have to do: fit another regression

$$Y_i \;=\; \alpha_3^* + \beta_3^* T_i + \epsilon_{3i}^*$$

in addition to the previous two regressions:

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i} \end{aligned}$$

- Estimated causal mediation effects as a function of ρ (and identifiable parameters)

Theorem 3 (Identification with a Given Error Correlation)

Under Assumption 3,

$$\bar{\delta}(0) \;=\; \bar{\delta}(1) \;=\; \beta_2 \left(\frac{\sigma_{23}^*}{\sigma_2^2} - \frac{\rho}{\sigma_2} \sqrt{\frac{1}{1-\rho^2}\left({\sigma_3^*}^2 - \frac{{\sigma_{23}^*}^2}{\sigma_2^2} \right)} \right),$$

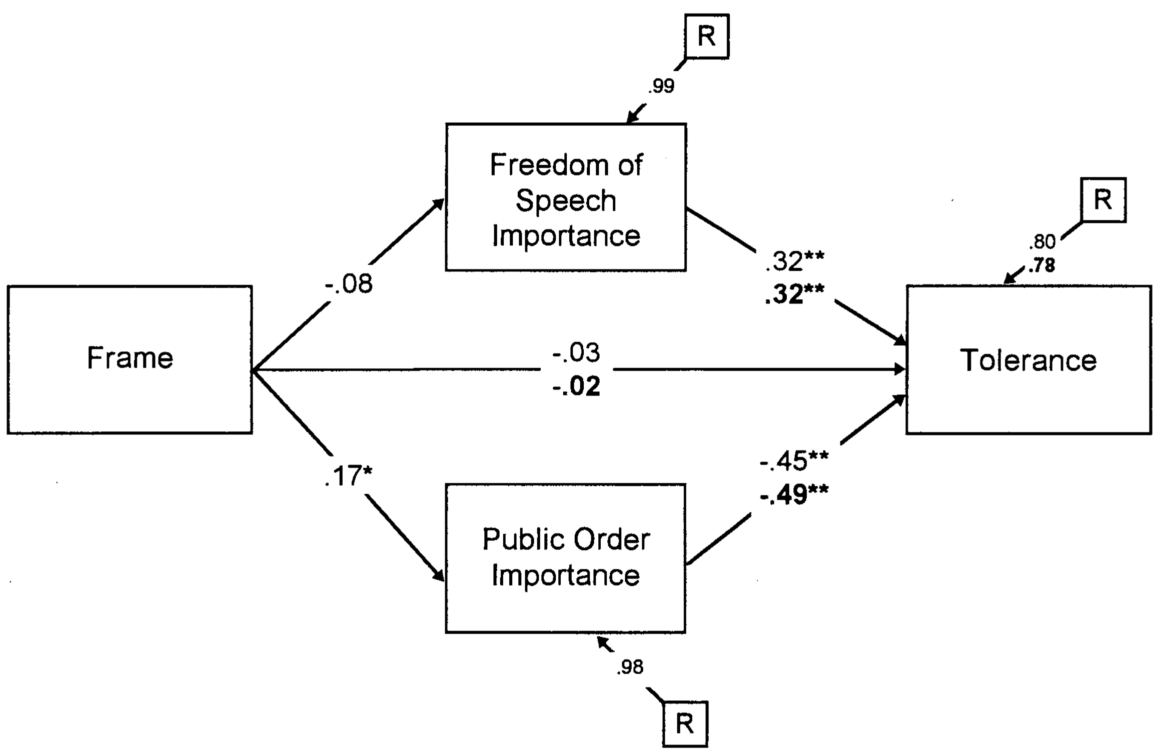
where $\sigma_j^2 \equiv \mathrm{Var}(\epsilon_{ji})$ *for* $j = 2, 3$, ${\sigma_3^*}^2 \equiv \mathrm{Var}(\epsilon_{3i}^*)$, $\sigma_{23}^* \equiv \mathrm{Cov}(\epsilon_{2i}, \epsilon_{3i}^*)$, *and* $\epsilon_{3i}^* = \gamma \epsilon_{2i} + \epsilon_{3i}$.

- When do my results go away completely?
- $\bar{\delta}(t) = 0$ if and only if $\rho = \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i}^*)$ (easy to compute!)

Political Psychology Experiment: Nelson *et al.* (*APSR*)

- How does media framing affect citizens' political opinions?
- News stories about the Ku Klux Klan rally in Ohio
- Free speech frame ($T_i = 0$) and public order frame ($T_i = 1$)
- Randomized experiment with the sample size $= 136$

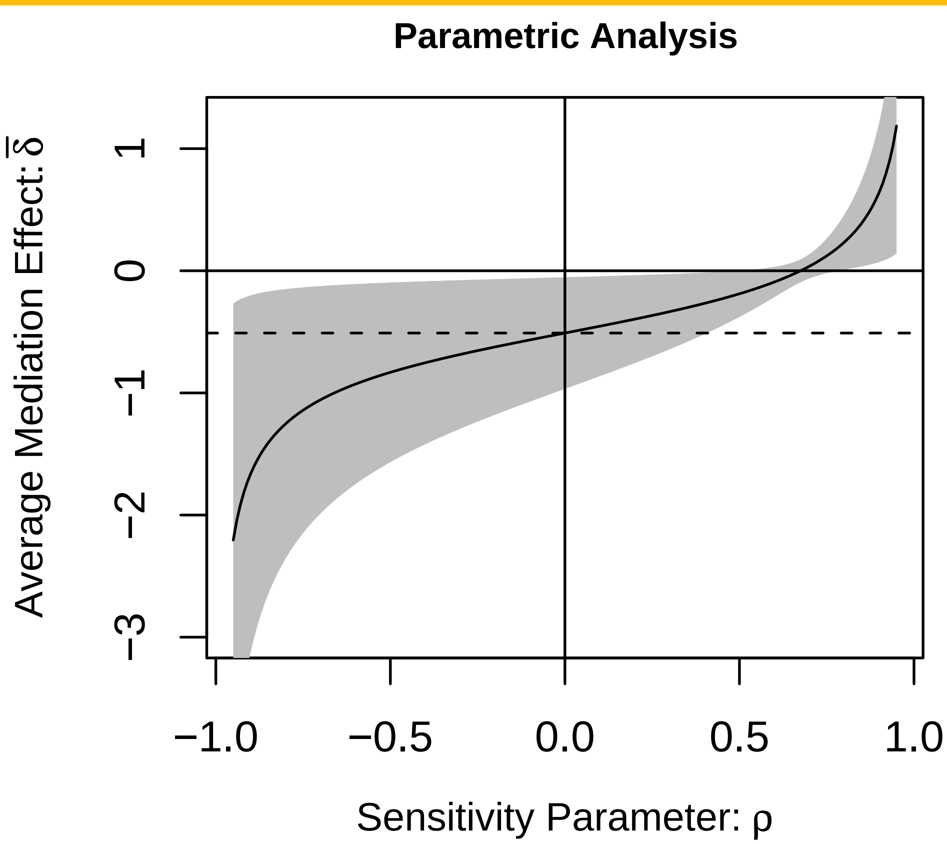
- Mediators: general attitudes (12 point scale) about the importance of free speech and public order
- Outcome: tolerance (7 point scale) for the Klan rally
- Expected findings: negative mediation effects



Analysis under Sequential Ignorability

Estimator	Mediator: Public Order	Mediator: Free Speech
Parametric		
No-interaction	-0.510	-0.126
	$[-0.969, -0.051]$	$[-0.388, 0.135]$
$\hat{\delta}(0)$	-0.451	-0.131
	$[-0.871, -0.031]$	$[-0.404, 0.143]$
$\hat{\delta}(1)$	-0.566	-0.122
	$[-1.081, -0.050]$	$[-0.380, 0.136]$
Nonparametric		
$\hat{\delta}(0)$	-0.374	-0.094
	$[-0.823, 0.074]$	$[-0.434, 0.246]$
$\hat{\delta}(1)$	-0.596	-0.222
	$[-1.168, -0.024]$	$[-0.662, 0.219]$

Parametric Sensitivity Analysis



Concluding Remarks

- Quantitative analysis can be used to identify causal mechanisms!
- Estimate causal mediation effects rather than marginal effects
- Wide applications in social science disciplines