

# Causal Mediation Q&A

## Use of Directed Acyclic Graphs (DAGs) and Potential Outcomes in Social Science Research

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# Do experiments have any value without mediation?

- Yes, but it is crucial to understand mechanisms:
  - scientists want to test theories which are about mechanisms
  - policy makers want to devise better policies
  - understanding of mechanisms  $\rightsquigarrow$  external validity
- Two ways to address the question, “why does a treatment work?”
  - 1 mediation  $\rightsquigarrow$  causal process
  - 2 interaction  $\rightsquigarrow$  causal components

# What do you think about mechanism experiments?

- “mechanism experiments” (Ludwig, Kling, and Mullainathan, 2011)
- “causal chain approach” (Spencer, Zanna, and Fong, 2005)
  - ① Randomize  $T$  to identify its effect on  $Y$  and its effect on  $M$
  - ② Randomize  $M$  to identify its effect on  $Y$
- This is certainly a progress towards understanding mechanisms
- Two issues with this approach (Imai, Tingley, and Yamamoto, JRSSA, 2013):
  - ① Effects of direct manipulation of  $M$  may differ from those of “natural” change in  $M$  induced by  $T$
  - ② Effect heterogeneity: even if the average effect of  $T$  on  $M$  and that of  $M$  on  $Y$  are both positive, the average mediation effect of  $T$  on  $Y$  can be negative

# How sensitive do the results of sensitivity analysis have to be before doubting mediation analysis?

- What sensitivity analysis provides: the amount of hidden bias that makes one's mediational results go away
- Traditional tests: sampling uncertainty of one's mediational effects that are assumed to be identifiable with the infinite amount of data
- Can a scientific community agree on the required degree of sensitivity?  $\rightsquigarrow$  maybe not
- Rosenbaum's example:
  - ① Effect of smoking on cancer:  $\Gamma = 6$
  - ② Effect of coffee on myocardial infarction:  $\Gamma = 1.3$
- Need to accumulate sensitivity analysis results
- Need to look for confounders that reduce sensitivity

# Other Questions

- 1 Why can't we just show those who have the large effects of  $T$  on  $M$  also exhibit the large effects of  $M$  on  $Y$ ?
  - Yes, but those effects must be identified
  - Reducing heterogeneity helps the identification of mediation effects
- 2 Is mediation analysis uninformative because it can hardly be definitive?
  - No. Almost no scientific study can be definitive.
  - But mediation is about purely counterfactual quantities
- 3 What researchers can do to maximize the plausibility of sequential ignorability?
  - Better design with clever manipulation of mediators
  - Importance of sensitivity analysis

# DAGs vs. Potential Outcomes

- Directed Acyclic Graphs (DAGs): Spirtes, Lauritzen, Pearl, etc.
- Potential outcomes: Neyman, Rubin, Holland, etc.
- Thus far, social scientists have used the potential outcomes framework more extensively than DAGs
- New textbook by Imbens and Rubin (2015):

*Pearl's work is interesting, and many researchers find his arguments that path diagrams are a natural and convenient way to express assumptions about causal structures appealing. In our own work, perhaps influenced by the type of examples arising in social and medical sciences, we have not found this approach to aid drawing of causal inferences.*

## Judea's Blog Post: "Are economists smarter than epidemiologists?"

*So, what is it about epidemiologists that drives them to seek the light of new tools, while economists seek comfort in partial blindness, while missing out on the causal revolution? Can economists do in their heads what epidemiologists observe in their graphs? Can they, for instance, identify the testable implications of their own assumptions? Can they decide whether the IV assumptions are satisfied in their own models of reality? Of course they can't; such decisions are intractable to the graph-less mind.*

# My Personal Experience and View

- I have been using potential outcomes in most of my research, but recently I have started using DAGs
- Potential outcomes are useful when thinking about treatment assignment mechanism  $\rightsquigarrow$  experiments, quasi-experiments
- DAGs are useful when thinking about the entire causal structure  $\rightsquigarrow$  complex causal relationships
- Both are better suited for causal inference than the standard regression framework

# Causal Mediation Analysis: Potential Outcomes

- Sequential ignorability (Imai, Keele, and Yamamoto, 2010):

$$\{Y_i(t', m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i = x \quad (1)$$

$$Y_i(t', m) \perp\!\!\!\perp M_i(t) \mid T_i = t, X_i = x \quad (2)$$

for all  $t, t'$ , and  $x$

- Interpretation: “as-if random” treatment assignment

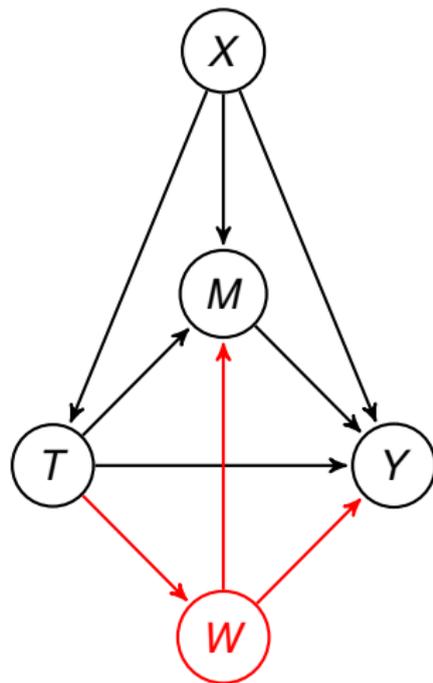
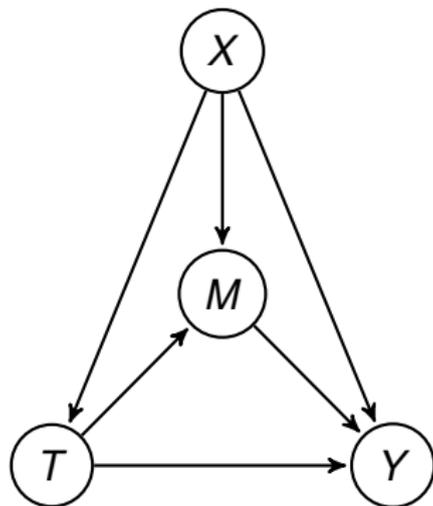
①  $T$  is as-if random given  $X$

②  $M$  is as-if random given  $T$  and  $X$

- No post-treatment confounder  $\rightsquigarrow$  only conditioning on  $\{T, X\}$  in (2)

$$Y_i(t', m) \perp\!\!\!\perp M_i(t) \mid W_i(t) = w, T_i = t, X_i = x$$

# Causal Mediation Analysis: DAGs



- The **absence** of arrows and nodes implies the assumptions
- DAGs help establish identification in more complicated situations

# Linear Regression with Unit Fixed Effects

- Balanced panel data with  $N$  units and  $T$  time periods
- $Y_{it}$ : outcome variable
- $X_{it}$ : causal or treatment variable of interest
- Model:

$$Y_{it} = \alpha_i + \beta X_{it} + \epsilon_{it}$$

where  $\alpha_i = h(\mathbf{U}_i)$  and  $\mathbf{U}_i$  represents unobserved time-invariant confounders

- Standard assumption: **Strict exogeneity**

$$\mathbb{E}(\epsilon_{it} \mid \mathbf{X}_i, \alpha_i) = 0$$

where  $\mathbf{X}_i$  is a  $T \times 1$  vector of treatment variables for unit  $i$

# Fixed Effects: Potential Outcomes

- 1 Treatments do not directly affect future outcomes

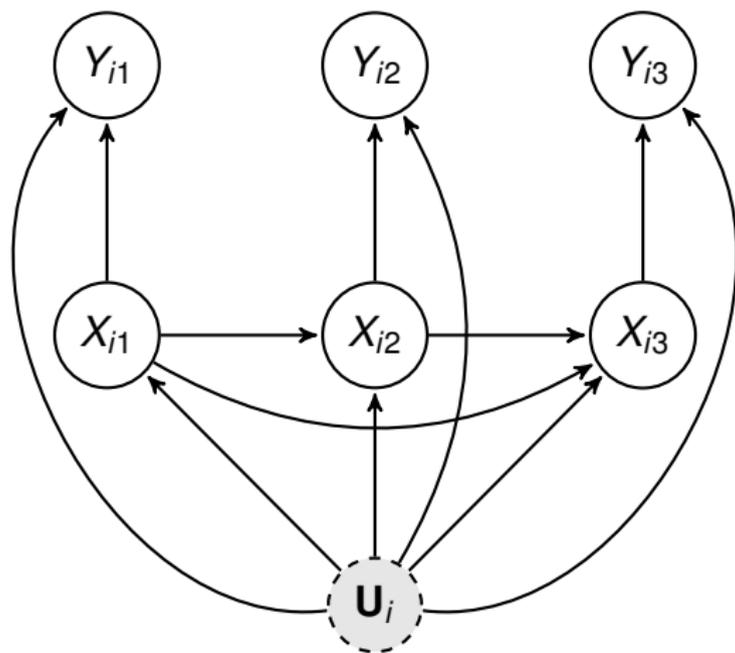
$$Y_{it}(X_{i1}, X_{i2}, \dots, X_{i,t-1}, X_{it}) = Y_{it}(X_{it})$$

- 2 Sequential ignorability:

$$\begin{aligned} \{Y_{it}(1), Y_{it}(0)\}_{t=1}^T &\perp\!\!\!\perp X_{i1} \mid \mathbf{U}_i \\ &\vdots \\ \{Y_{it}(1), Y_{it}(0)\}_{t=1}^T &\perp\!\!\!\perp X_{it'} \mid X_{i1}, \dots, X_{i,t'-1}, \mathbf{U}_i \\ &\vdots \\ \{Y_{it}(1), Y_{it}(0)\}_{t=1}^T &\perp\!\!\!\perp X_{iT} \mid X_{i1}, \dots, X_{i,T-1}, \mathbf{U}_i \end{aligned}$$

“as-if random” treatment assignment without conditioning on the previous outcomes

# Fixed Effects: DAG



# Fixed Effects: DAG

