Matching Methods for Causal Inference with Time-Series Cross-Sectional Data

Kosuke Imai

Department of Government and Department of Statistics
Harvard University

20th Anniversary Conference for the Center for Statistics in Social Sciences
University of Washington

May 24, 2019
Joint work with In Song Kim (MIT) and Erik Wang (Princeton)
Data
- Past: government data, national survey data
- Today: more of old types of data and lots of new data
  - Randomized experiments and surveys conducted by researchers
  - Administration records: voter files, contributions, lobbying, . . .
  - Social media data: websites, blogs, tweets, cell phones, . . .
  - GIS data: satellite, climate, natural resource discoveries, . . .
  - Text, image, audio data: news, speeches, bills, commercials, . . .

Methods
- Past: linear and other parametric regressions
- Today: regressions and variety of advanced methodologies
  - Causal inference with experimental and observational data
  - Bayes and machine learning for structured and unstructured data

Society
- Past: little interests in quantitative social science
- Today: huge interests among students, NGOs, journalists, courts, government agencies, international organizations, etc.
Motivation and Overview

- Matching methods have become part of toolkit for social scientists
  1. reduces model dependence in observational studies
  2. provides diagnostics through balance checks
  3. clarifies comparison between treated and control units

- Almost all existing matching methods deal with cross-sectional data

- We propose a matching method for time-series cross-sectional data
  1. create a matched set for each treated observation
  2. refine the matched set via any matching or weighting method
  3. compute the difference-in-differences estimator

- Provide a model-based standard error

- Develop an open-source software package PanelMatch

- Empirical applications:
  - Democracy and economic growth (Acemoglu et al.)
  - Interstate war and inheritance tax (Scheve & Stasavage)
Democracy and Economic Growth

- Acemoglu et al. (2017): an up-to-date empirical study of the long-standing question in political economy
- TSCS data set: 184 countries from 1960 to 2010
- Dynamic linear regression model with fixed effects:

\[
Y_{it} = \alpha_i + \gamma_t + \beta X_{it} + \sum_{\ell=1}^{4} \left\{ \rho_{\ell} Y_{i,t-\ell} + \zeta_{\ell}^{\top} Z_{i,t-\ell} \right\} + \epsilon_{it}
\]

- \(X_{it}\): binary democracy indicator
- \(Y_{it}\): log real GDP per capita
- \(Z_{it}\): time-varying covariates (population, trade, social unrest, etc.)

- Sequential exogeneity assumption:

\[
E(\epsilon_{it} \mid \{Y_{it'}\}_{t'=1}^{t-1}, \{X_{it'}\}_{t'=1}^{t}, \{Z_{it'}\}_{t'=1}^{t-1}, \alpha_i, \gamma_t) = 0
\]

- Nickell bias \(\rightsquigarrow\) GMM estimation with instruments (Arellano & Bond)
### Regression Results

<table>
<thead>
<tr>
<th></th>
<th>Estimate (SE)</th>
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<th>Estimate (SE)</th>
<th>Estimate (SE)</th>
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<tr>
<td><strong>ATE ((\hat{\beta}))</strong></td>
<td>0.787 (0.226)</td>
<td>0.875 (0.374)</td>
<td>0.666 (0.307)</td>
<td>0.917 (0.461)</td>
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<td>(\hat{\rho}_1)</td>
<td>1.238 (0.038)</td>
<td>1.204 (0.041)</td>
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<td>(\hat{\rho}_2)</td>
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<td>-0.133 (0.041)</td>
<td>-0.121 (0.038)</td>
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<td>-0.028 (0.028)</td>
<td>0.005 (0.030)</td>
<td>0.014 (0.029)</td>
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<td>(\hat{\rho}_4)</td>
<td>-0.043 (0.017)</td>
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<td>-0.018 (0.023)</td>
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<td>Yes</td>
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<td>GMM</td>
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<td>GMM</td>
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<td><strong>N</strong></td>
<td>6,336</td>
<td>4,416</td>
<td>6,161</td>
<td>4,245</td>
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</table>
Regression models does not tell us where the variation comes from.

Estimation of counterfactual outcomes requires comparison between treated and control observations.

Identification strategy:
- within-unit over-time variation
- within-time across-units variation
Quantity of Interest and Assumptions

- Choose number of lags \( L = 2, \ldots \), for confounder adjustment
- Choose number of leads, \( F = 0, 1, \ldots \), for short or long term effects
- Average Treatment Effect of Policy Change for the Treated (ATT):

\[
\mathbb{E} \left\{ Y_{i,t+F} \left( X_{it} = 1, X_{i,t-1} = 0, \{ X_{i,t-\ell} \}_{\ell=2}^{L} \right) - Y_{i,t+F} \left( X_{it} = 0, X_{i,t-1} = 0, \{ X_{i,t-\ell} \}_{\ell=2}^{L} \right) \mid X_{it} = 1, X_{i,t-1} = 0 \right\}
\]

- Assumptions:
  1. No spillover effect
  2. Limited carryover effect (up to \( L \) time periods)
  3. Parallel trend after conditioning:

\[
\mathbb{E} \left[ Y_{i,t+F} \left( X_{it} = X_{i,t-1} = 0, \{ X_{i,t-\ell} \}_{\ell=2}^{L} \right) - Y_{i,t-1} \mid X_{it} = 1, X_{i,t-1} = 0, \{ X_{i,t-\ell}, Y_{i,t-\ell} \}_{\ell=2}^{L}, \{ Z_{i,t-\ell} \}_{\ell=0}^{L} \right] = \mathbb{E} \left[ Y_{i,t+F} \left( X_{it} = X_{i,t-1} = 0, \{ X_{i,t-\ell} \}_{\ell=2}^{L} \right) - Y_{i,t-1} \mid X_{it} = 0, X_{i,t-1} = 0, \{ X_{i,t-\ell}, Y_{i,t-\ell} \}_{\ell=2}^{L}, \{ Z_{i,t-\ell} \}_{\ell=0}^{L} \right]
\]
Constructing Matched Sets

- Control units with identical treatment history from time $t - L$ to $t - 1$
- Construct a matched set for each treated observation
- Formal definition:

$$
\mathcal{M}_{it} = \{i' : i' \neq i, X_{i't} = 0, X_{i't'} = X_{it'} \text{ for all } t' = t - 1, \ldots, t - L\}
$$

- Some treated observations have no matched control
  \(\Rightarrow\) change the quantity of interest by dropping them
- Similar to the risk set of Li et al. (2001) but we do not exclude those who already receive the treatment
### An Example of Matched Set

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Democracy</th>
<th>logGDP</th>
<th>Population</th>
<th>Trade</th>
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</table>
Refining Matched Sets

- Make additional adjustments for past outcomes and confounders
- Use any matching or weighting method
- Mahalanobis distance matching:
  1. Compute the distance between treated and matched control obs.

\[ S_{it}(i') = \frac{1}{L} \sum_{\ell=1}^{L} \sqrt{\left( V_{i,t-\ell} - V_{i',t-\ell} \right)^{\top} \Sigma_{i,t-\ell}^{-1} \left( V_{i,t-\ell} - V_{i',t-\ell} \right)} \]

where \( V_{it'} = (Y_{it'}, Z_{it',t'+1})^{\top} \) and \( \Sigma_{it'} = \text{Cov}(V_{it'}) \)
  2. Match the most similar \( J \) matched control observations

- Propensity score weighting:
  4. Estimate the propensity score

\[ e_{it}(\{V_{i,t-\ell}\}_{\ell=1}^{L}) = \Pr(X_{it} = 1 \mid \{V_{i,t-\ell}\}_{\ell=1}^{L}) \]
  2. Weight each matched control observation by its inverse
## An Example of Refinement

<table>
<thead>
<tr>
<th>Country</th>
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</tbody>
</table>
The Multi-period Difference-in-Differences Estimator

- Compute the weighted average of difference-in-differences among matched control observations
- Weights are based on refinement $\sim$ marginal structural models for the long-term effect of a fixed treatment sequence
- A synthetic control for each treated observation

The Multi-period DiD estimator:

$$\frac{1}{\sum_{i=1}^{N} \sum_{t=L+1}^{T-F} D_{it}} \sum_{i=1}^{N} \sum_{t=L+1}^{T-F} D_{it} \left\{ (Y_{i,t+F} - Y_{i,t-1}) - \sum_{i' \in M_{it}} w_{it}^i (Y_{i',t+F} - Y_{i',t-1}) \right\}$$

- Equivalent to the weighted two-way fixed effects estimator:

$$\arg\min_{\beta} \sum_{i=1}^{N} \sum_{t=1}^{T} W_{it} \left\{ (Y_{it} - \overline{Y}_i - \overline{Y}_t + \overline{Y}^*) - \beta (X_{it} - \overline{X}_i - \overline{X}_t + \overline{X}^*) \right\}^2$$

where $(\overline{Y}_i^*, \overline{Y}_t^*, \overline{Y}^*)$ and $(\overline{X}_i^*, \overline{X}_t^*, \overline{X}^*)$ are weighted averages
Checking Covariate Balance and Computing Standard Error

- Balance for covariate $j$ at time $t - \ell$ in each matched set:

$$B_{it}(j, \ell) = \frac{V_{i,t-\ell,j} - \sum_{i' \in M_{it}} w_{it}' V_{i',t-\ell,j}}{\sqrt{\frac{1}{N_1-1} \sum_{i'=1}^{N} \sum_{t' = L+1}^{T-F} D_{it'} (V_{i',t'-\ell,j} - \overline{V}_{t'-\ell,j})^2}}$$

- Average this measure across all treated observations:

$$\overline{B}(j, \ell) = \frac{1}{N_1} \sum_{i=1}^{N} \sum_{t=L+1}^{T-F} D_{it} B_{it}(j, \ell)$$

- Standard error calculation $\leadsto$ consider weight as a covariate

1. Block bootstrap
2. Model-based cluster robust standard error within the GMM framework
Empirical Application

- ATT with $L = 4$ and $F = 1, 2, 3, 4$
- We consider democratization and authoritarian reversal
- Examine the number of matched control units
- 18 (13) treated observations have no matched control
Improved Covariate Balance

Before Matching

Before Refinement

Mahalanobis Distance Matching

Propensity Score Matching

Propensity Score Weighting

Imai, Kim, and Wang (HU/MIT/PU)  Matching for TCS Data  UW, CSSS (May 24, 2019)
Estimated Causal Effects

Mahalanobis Matching
Up to 5 matches
Up to 10 matches

Propensity Score Matching
Up to 5 matches
Up to 10 matches

Propensity Score Weighting

Estimated Effect of Democratization

Estimated Effect of Authoritarian Reversal

Treatment reversal allowed
Treatment reversal not allowed

Years relative to the administration of treatment

Imai, Kim, and Wang (HU/MIT/PU)
Concluding Remarks

- Matching as transparent and simple methods for causal inference
- Yet, matching has not been applied to time-series cross-sectional data

- We propose a matching framework for TSCS data
  1. construct matched sets
  2. refine matched sets
  3. compute difference-in-differences estimator

- Checking covariates and computing standard errors
- R package PanelMatch implements all of these methods

- Ongoing research: addressing possible spillover effects
Available papers:


Send comments and suggestions to:

Imai@Harvard.Edu

More information about this and other research:

https://imai.fas.harvard.edu/