Statistical Analysis of Causal Mechanisms

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Statistics and Causal Mechanisms

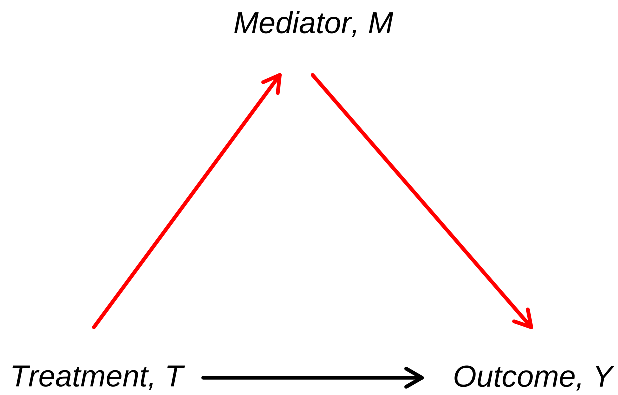
- Causal inference as central goal of social science
- Challenge is how to identify causal mechanism

- Even randomized experiments can only determine *whether* the treatment causes changes in the outcome
- Not *how* and *why* the treatment affects the outcome
- Qualitative research uses process tracing

- How can quantitative research be used to identify causal mechanisms?

Overview of the Talk

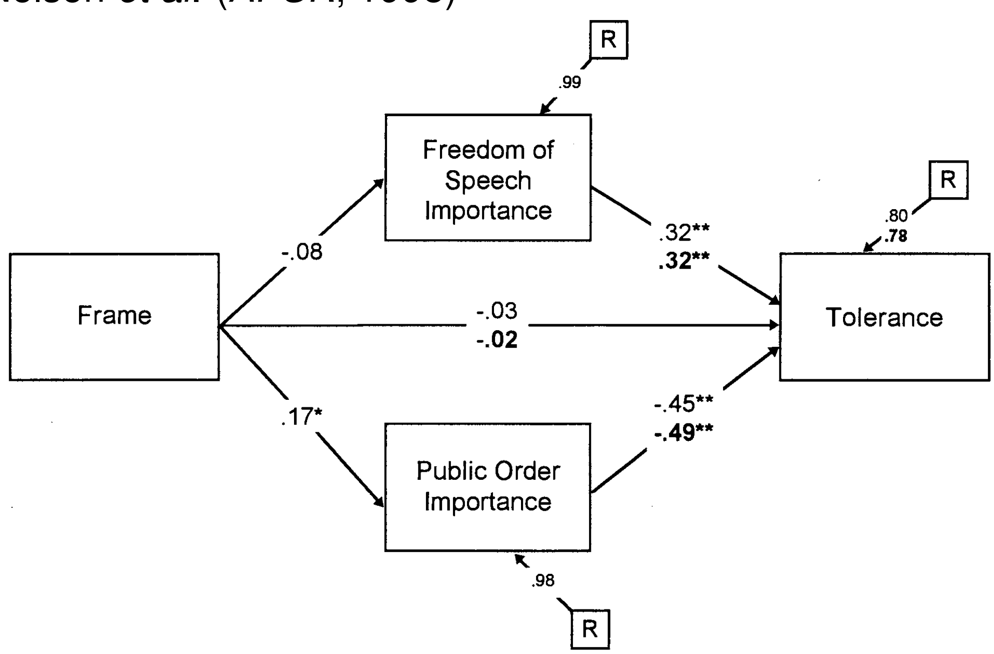
- **Goal:** Convince you that statistics *can* play a role in identifying causal mechanisms
- **Method:** Causal Mediation Analysis



- Direct and indirect effects; intermediate and intervening variables

Causal Mediation Analysis in American Politics

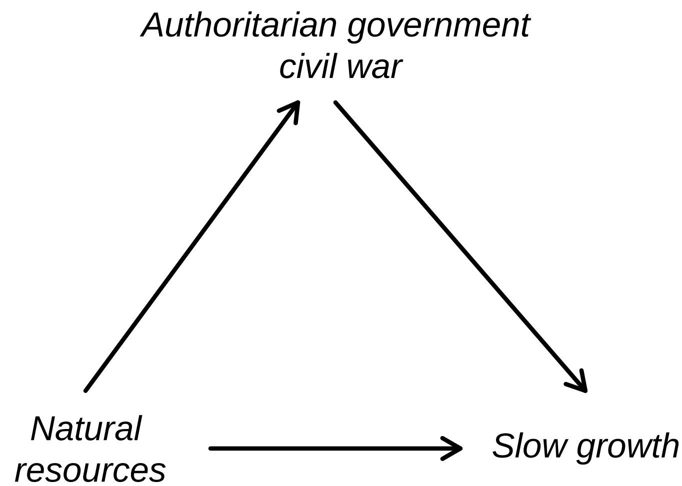
- The political psychology literature on media framing
- Nelson *et al*. (*APSR*, 1998)



- Popular in social psychology

Causal Mediation Analysis in Comparative Politics

- Resource curse thesis



- Causes of civil war: Fearon and Laitin (*APSR*, 2003)

Causal Mediation Analysis in International Relations

- The literature on international regimes and institutions
- Krasner (*International Organization*, 1982)

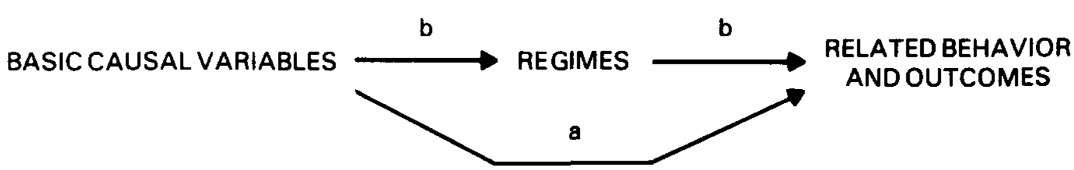


Figure 2

- Power and interests are mediated by regimes

Current Practice in the Discipline

- Regression
$$Y_i = \alpha + \beta T_i + \gamma M_i + \delta X_i + \epsilon_i$$
- Each coefficient is interpreted as a causal effect
- Sometimes, it's called marginal effect
- Idea: increase T_i by one unit while holding M_i and X_i constant

- The Problem (Post-treatment bias): if you change T_i, that may also change M_i
- Usual advice: only include causally prior (or pre-treatment) variables
- But, then you lose causal mechanisms!

Formal Statistical Framework of Causal Inference

- Units: $i = 1, \ldots, n$
- "Treatment": $T_i = 1$ if treated, $T_i = 0$ otherwise
- *Observed* outcome: Y_i
- Pre-treatment covariates: X_i
- Potential outcomes: $Y_i(1)$ and $Y_i(0)$ where $Y_i = Y_i(T_i)$

Voters	Contact	Turnout		Age	Party ID
i	T_i	$Y_i(1)$	$Y_i(0)$	X_i	X_i
1	1	1	?	20	D
2	0	?	0	55	R
3	0	?	1	40	R
⋮	⋮	⋮	⋮	⋮	⋮
n	1	0	?	62	D

- Causal effect: $Y_i(1) - Y_i(0)$

Identification of Causal Effects in Standard Settings

- Average Treatment Effect (ATE): $\tau \equiv \mathbb{E}(Y_i(1) - Y_i(0))$
- **Randomized experiments**:
  - Randomization of the treatment: $(Y_i(1), Y_i(0)) \perp\!\!\!\perp T_i$
  - Identification:

$$\tau = \mathbb{E}(Y_i \mid T_i = 1) - \mathbb{E}(Y_i \mid T_i = 0)$$

- **Observational studies**:
  - No omitted variables (ignorability): $(Y_i(1), Y_i(0)) \perp\!\!\!\perp T_i \mid X_i$
  - Identification:

$$\tau = \mathbb{E}(Y_i \mid T_i = 1, X_i) - \mathbb{E}(Y_i \mid T_i = 0, X_i)$$

  - Relationship with the regression:

$$Y_i(T_i) = \alpha + \beta T_i + \gamma X_i + \epsilon_i$$

    where the assumption implies $T_i \perp\!\!\!\perp \epsilon_i \mid X_i$

Notation for Causal Mediation Analysis

- Binary treatment: $T_i \in \{0, 1\}$
- Mediator: M_i
- Outcome: Y_i
- Observed covariates: X_i
- Potential mediators: $M_i(t)$ where $M_i = M_i(T_i)$
- Potential outcomes: $Y_i(t, m)$ where $Y_i = Y_i(T_i, M_i(T_i))$

Defining and Interpreting Causal Mediation Effects

- Total causal effect: $\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0))$
- Causal mediation effects:

$$\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Change the mediator from $M_i(0)$ to $M_i(1)$ while holding the treatment constant at t
- Indirect effect of the treatment on the outcome through the mediator under treatment status t
- $Y_i(t, M_i(t))$ is observable but $Y_i(t, M_i(1-t))$ is not

- Direct effects:

$$\zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))$$

- Change the treatment from 0 to 1 while holding the mediator constant at $M_i(t)$
- Total effect = mediation (indirect) effect + direct effect:

$$\tau_i = \delta_i(t) + \zeta_i(1-t)$$

- Quantities of interest: Average Causal Mediation Effects,

$$\bar{\delta}(t) \equiv \mathbb{E}(\delta_i(t)) = \mathbb{E}\{Y_i(t, M_i(1)) - Y_i(t, M_i(0))\}$$

The Main Identification Result

Assumption 1 (Sequential Ignorability)

$$\{Y_i(t, m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i,$$
$$Y_i(t, m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

for $t = 0, 1$

- Existing statistics literature concludes that an additional assumption is required for the identification of mediation effects
- However, we show that sequential ignorability *alone* is sufficient
- Propose a nonparametric estimator and derive its asymptotic variance
- No functional and distributional assumption is required

Identification under Linear Structural Equation Model

Theorem 1 (Identification under LSEM)

Consider the following linear structural equation model

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i}, \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i}. \end{aligned}$$

Under Assumption 1, the average causal mediation effects are identified as $\bar{\delta}(0) = \bar{\delta}(1) = \beta_2\gamma$.

- Run two regressions and multiply two coefficients!
- Direct effect: β_3
- Total effect: $\beta_2\gamma + \beta_3$
- If regressions are not linear (e.g., probit), then more complicated but can be done

Need for Sensitivity Analysis

- The sequential ignorability assumption is often too strong
- Need to assess the robustness of findings via sensitivity analysis
- Question: How large a departure from the key assumption must occur for the conclusions to no longer hold?
- Parametric and nonparametric sensitivity analysis by assuming

$$\{Y_i(t,m), M_i(t)\} \perp\!\!\!\perp T_i \mid X_i$$

but not

$$Y_i(t,m) \perp\!\!\!\perp M_i \mid T_i, X_i$$

Parametric Sensitivity Analysis

- Sensitivity parameter: $\rho \equiv \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i})$
- Existence of omitted variables leads to non-zero ρ
- Set ρ to different values and see how mediation effects change
- All you have to do: fit another regression

$$Y_i = \alpha_3^* + \beta_3^* T_i + \epsilon_{3i}^*$$

in addition to the previous two regressions:

$$\begin{aligned} M_i &= \alpha_2 + \beta_2 T_i + \epsilon_{2i} \\ Y_i &= \alpha_3 + \beta_3 T_i + \gamma M_i + \epsilon_{3i} \end{aligned}$$

- Estimated causal mediation effects as a function of ρ (and identifiable parameters)

Theorem 2 (Identification with a Given Error Correlation)

Under Assumption 3,

$$\bar{\delta}(0) = \bar{\delta}(1) = \beta_2 \left(\frac{\sigma_{23}^*}{\sigma_2^2} - \frac{\rho}{\sigma_2} \sqrt{\frac{1}{1-\rho^2} \left(\sigma_3^{*2} - \frac{\sigma_{23}^{*\,2}}{\sigma_2^2} \right)} \right),$$

where $\sigma_j^2 \equiv \mathrm{Var}(\epsilon_{ji})$ *for* $j = 2, 3$, $\sigma_3^{*2} \equiv \mathrm{Var}(\epsilon_{3i}^*)$, $\sigma_{23}^* \equiv \mathrm{Cov}(\epsilon_{2i}, \epsilon_{3i}^*)$, *and* $\epsilon_{3i}^* = \gamma\epsilon_{2i} + \epsilon_{3i}$.

- When do my results go away completely?
- $\bar{\delta}(t) = 0$ if and only if $\rho = \mathrm{Corr}(\epsilon_{2i}, \epsilon_{3i}^*)$ (easy to compute!)

Political Psychology Experiment: Nelson *et al.* (*APSR*)

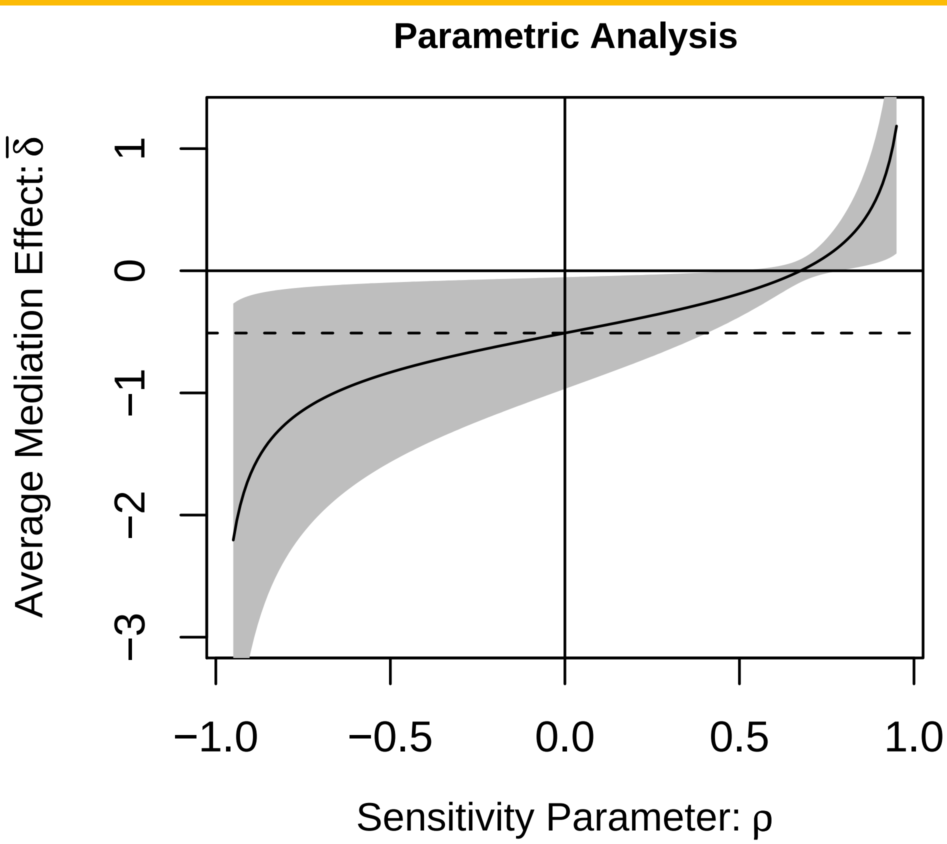
- How does media framing affect citizens' political opinions?
- News stories about the Ku Klux Klan rally in Ohio
- Free speech frame ($T_i = 0$) and public order frame ($T_i = 1$)
- Randomized experiment with the sample size $= 136$

- Mediators: general attitudes (12 point scale) about the importance of free speech and public order
- Outcome: tolerance (7 point scale) for the Klan rally
- Expected findings: negative mediation effects

Analysis under Sequential Ignorability

Estimator	Mediator: Public Order	Mediator: Free Speech
Parametric		
No-interaction	-0.510	-0.126
	$[-0.969, -0.051]$	$[-0.388, 0.135]$
$\hat{\delta}(0)$	-0.451	-0.131
	$[-0.871, -0.031]$	$[-0.404, 0.143]$
$\hat{\delta}(1)$	-0.566	-0.122
	$[-1.081, -0.050]$	$[-0.380, 0.136]$
Nonparametric		
$\hat{\delta}(0)$	-0.374	-0.094
	$[-0.823, 0.074]$	$[-0.434, 0.246]$
$\hat{\delta}(1)$	-0.596	-0.222
	$[-1.168, -0.024]$	$[-0.662, 0.219]$

Parametric Sensitivity Analysis



Concluding Remarks and Future Work

- Quantitative analysis can be used to identify causal mechanisms!
- Estimate causal mediation effects rather than marginal effects
- Wide applications in social science disciplines

- Contribution to statistical literature:
  1. Clarify assumptions
  2. Extend parametric method
  3. Develop nonparametric method
  4. Provide new sensitivity analysis