Covariate Balancing Propensity Score

Kosuke Imai

Princeton University

Talk at Yokohama City University

July 3, 2017

Joint work with Christian Fong, Chad Hazlett, and Marc Ratkovic

Motivation and Overview

- Central role of propensity score in causal inference
  - Adjusting for observed confounding in observational studies
  - Generalizing experimental and instrumental variables estimates
- Propensity score tautology
  - sensitivity to model misspecification
  - adhoc specification searches
- Covariate Balancing Propensity Score (CBPS)
  - Estimate the propensity score such that covariates are balanced
  - Inverse probability weights for marginal structural models
- Extensions:
  1. Continuous treatment (with Christian Fong and Chad Hazlett)
  2. Time-varying treatments (with Marc Ratkovic)
  3. High dimensional covariates (with Yang Ning and Sida Peng)

Propensity Score

- Notation:
  - $T_i \in \{0, 1\}$: binary treatment
  - X_i: pre-treatment covariates
- Dual characteristics of propensity score:
  1. Predicts treatment assignment:

$$\pi(X_i) \;=\; \Pr(T_i = 1 \mid X_i)$$

  2. Balances covariates (Rosenbaum and Rubin, 1983):

$$T_i \perp\!\!\!\perp X_i \;\mid\; \pi(X_i)$$

- But, propensity score must be estimated (more on this later)

Use of Propensity Score for Causal Inference

- Matching
- Subclassification
- Weighting (Horvitz-Thompson):

$$\frac{1}{n}\sum_{i=1}^{n}\left\{\frac{T_i Y_i}{\hat{\pi}(X_i)} - \frac{(1-T_i)Y_i}{1-\hat{\pi}(X_i)}\right\}$$

  where weights are often normalized
- Doubly-robust estimators (Robins *et al.*):

$$\frac{1}{n}\sum_{i=1}^{n}\left[\left\{\hat{\mu}(1,X_i) + \frac{T_i(Y_i - \hat{\mu}(1,X_i))}{\hat{\pi}(X_i)}\right\} - \left\{\hat{\mu}(0,X_i) + \frac{(1-T_i)(Y_i - \hat{\mu}(0,X_i))}{1-\hat{\pi}(X_i)}\right\}\right]$$

- They have become standard tools for applied researchers

Propensity Score Tautology

- Propensity score is unknown and must be estimated
  - Dimension reduction is purely theoretical: must model T_i given X_i
  - Diagnostics: covariate balance checking
- In theory: ellipsoidal covariate distributions
  $\Longrightarrow$ equal percent bias reduction
- In practice: skewed covariates and adhoc specification searches
- Propensity score methods are sensitive to model misspecification
- Tautology: propensity score methods only work when they work

Kang and Schafer (2007, *Statistical Science*)

- Simulation study: the deteriorating performance of propensity score weighting methods when the model is misspecified
- 4 covariates X_i^*: all are *i.i.d.* standard normal
- Outcome model: linear model
- Propensity score model: logistic model with linear predictors
- Misspecification induced by measurement error:
  - $X_{i1} = \exp(X_{i1}^*/2)$
  - $X_{i2} = X_{i2}^*/(1 + \exp(X_{1i}^*) + 10)$
  - $X_{i3} = (X_{i1}^* X_{i3}^*/25 + 0.6)^3$
  - $X_{i4} = (X_{i1}^* + X_{i4}^* + 20)^2$
- Four weighting estimators evaluated:
  1. Horvitz-Thompson (HT)
  2. Inverse-probability weighting with normalized weights (IPW)
  3. Weighted least squares regression (WLS)
  4. Doubly-robust least squares regression (DR)

Weighting Estimators Do Fine If the Model is Correct

Sample size	Estimator	Bias GLM	Bias True	RMSE GLM	RMSE True
(1) Both models correct					
$n = 200$	HT	0.33	1.19	12.61	23.93
	IPW	−0.13	−0.13	3.98	5.03
	WLS	−0.04	−0.04	2.58	2.58
	DR	−0.04	−0.04	2.58	2.58
$n = 1000$	HT	0.01	−0.18	4.92	10.47
	IPW	0.01	−0.05	1.75	2.22
	WLS	0.01	0.01	1.14	1.14
	DR	0.01	0.01	1.14	1.14
(2) Propensity score model correct					
$n = 200$	HT	−0.05	−0.14	14.39	24.28
	IPW	−0.13	−0.18	4.08	4.97
	WLS	0.04	0.04	2.51	2.51
	DR	0.04	0.04	2.51	2.51
$n = 1000$	HT	−0.02	0.29	4.85	10.62
	IPW	0.02	−0.03	1.75	2.27
	WLS	0.04	0.04	1.14	1.14
	DR	0.04	0.04	1.14	1.14

Weighting Estimators are Sensitive to Misspecification

		Bias		RMSE	
Sample size	Estimator	GLM	True	GLM	True
(3) Outcome model correct					
$n = 200$	HT	24.25	-0.18	194.58	23.24
	IPW	1.70	-0.26	9.75	4.93
	WLS	-2.29	0.41	4.03	3.31
	DR	-0.08	-0.10	2.67	2.58
$n = 1000$	HT	41.14	-0.23	238.14	10.42
	IPW	4.93	-0.02	11.44	2.21
	WLS	-2.94	0.20	3.29	1.47
	DR	0.02	0.01	1.89	1.13
(4) Both models incorrect					
$n = 200$	HT	30.32	-0.38	266.30	23.86
	IPW	1.93	-0.09	10.50	5.08
	WLS	-2.13	0.55	3.87	3.29
	DR	-7.46	0.37	50.30	3.74
$n = 1000$	HT	101.47	0.01	2371.18	10.53
	IPW	5.16	0.02	12.71	2.25
	WLS	-2.95	0.37	3.30	1.47
	DR	-48.66	0.08	1370.91	1.81

Covariate Balancing Propensity Score (CBPS)

- Idea: Estimate propensity score such that covariates are balanced
- Goal: Robust estimation of parametric propensity score model

- Covariate balancing conditions:

$$\mathbb{E}\left\{\frac{T_i X_i}{\pi_\beta(X_i)} - \frac{(1-T_i)X_i}{1-\pi_\beta(X_i)}\right\} = 0$$

- Optional over-identification via score conditions:

$$\mathbb{E}\left\{\frac{T_i \pi'_\beta(X_i)}{\pi_\beta(X_i)} - \frac{(1-T_i)\pi'_\beta(X_i)}{1-\pi_\beta(X_i)}\right\} = 0$$

- Can be interpreted as another covariate balancing condition

- Combine them with the Generalized Method of Moments

Revisiting Kang and Schafer (2007)

	Estimator	Bias GLM	Bias CBPS1	Bias CBPS2	Bias True	RMSE GLM	RMSE CBPS1	RMSE CBPS2	RMSE True
(1) Both models correct									
$n = 200$	HT	0.33	2.06	-4.74	1.19	12.61	4.68	9.33	23.93
	IPW	-0.13	0.05	-1.12	-0.13	3.98	3.22	3.50	5.03
	WLS	-0.04	-0.04	-0.04	-0.04	2.58	2.58	2.58	2.58
	DR	-0.04	-0.04	-0.04	-0.04	2.58	2.58	2.58	2.58
$n = 1000$	HT	0.01	0.44	-1.59	-0.18	4.92	1.76	4.18	10.47
	IPW	0.01	0.03	-0.32	-0.05	1.75	1.44	1.60	2.22
	WLS	0.01	0.01	0.01	0.01	1.14	1.14	1.14	1.14
	DR	0.01	0.01	0.01	0.01	1.14	1.14	1.14	1.14
(2) Propensity score model correct									
$n = 200$	HT	-0.05	1.99	-4.94	-0.14	14.39	4.57	9.39	24.28
	IPW	-0.13	0.02	-1.13	-0.18	4.08	3.22	3.55	4.97
	WLS	0.04	0.04	0.04	0.04	2.51	2.51	2.51	2.51
	DR	0.04	0.04	0.04	0.04	2.51	2.51	2.52	2.51
$n = 1000$	HT	-0.02	0.44	-1.67	0.29	4.85	1.77	4.22	10.62
	IPW	0.02	0.05	-0.31	-0.03	1.75	1.45	1.61	2.27
	WLS	0.04	0.04	0.04	0.04	1.14	1.14	1.14	1.14
	DR	0.04	0.04	0.04	0.04	1.14	1.14	1.14	1.14

CBPS Makes Weighting Methods Work Better

	Estimator	Bias GLM	Bias CBPS1	Bias CBPS2	Bias True	RMSE GLM	RMSE CBPS1	RMSE CBPS2	RMSE True
(3) Outcome model correct									
$n = 200$	HT	24.25	1.09	−5.42	−0.18	194.58	5.04	10.71	23.24
	IPW	1.70	−1.37	−2.84	−0.26	9.75	3.42	4.74	4.93
	WLS	−2.29	−2.37	−2.19	0.41	4.03	4.06	3.96	3.31
	DR	−0.08	−0.10	−0.10	−0.10	2.67	2.58	2.58	2.58
$n = 1000$	HT	41.14	−2.02	2.08	−0.23	238.14	2.97	6.65	10.42
	IPW	4.93	−1.39	−0.82	−0.02	11.44	2.01	2.26	2.21
	WLS	−2.94	−2.99	−2.95	0.20	3.29	3.37	3.33	1.47
	DR	0.02	0.01	0.01	0.01	1.89	1.13	1.13	1.13
(4) Both models incorrect									
$n = 200$	HT	30.32	1.27	−5.31	−0.38	266.30	5.20	10.62	23.86
	IPW	1.93	−1.26	−2.77	−0.09	10.50	3.37	4.67	5.08
	WLS	−2.13	−2.20	−2.04	0.55	3.87	3.91	3.81	3.29
	DR	−7.46	−2.59	−2.13	0.37	50.30	4.27	3.99	3.74
$n = 1000$	HT	101.47	−2.05	1.90	0.01	2371.18	3.02	6.75	10.53
	IPW	5.16	−1.44	−0.92	0.02	12.71	2.06	2.39	2.25
	WLS	−2.95	−3.01	−2.98	0.19	3.30	3.40	3.36	1.47
	DR	−48.66	−3.59	−3.79	0.08	1370.91	4.02	4.25	1.81

Propensity Score for a Continuous Treatment

- Standardize X_i and T_i such that
  - $\mathbb{E}(X_i^*) = \mathbb{E}(T_i^*) = \mathbb{E}(X_i^* T_i^*) = 0$
  - $\mathbb{V}(X_i) = \mathbb{V}(T_i) = 1$
- The stabilized weights:

$$w_i = \frac{f(T_i^*)}{f(T_i^* \mid X_i^*)}$$

- Standard approach (e.g., Robins *et al.* 2000):

$$\begin{aligned} T_i^* \mid X_i^* &\overset{\text{indep.}}{\sim} \mathcal{N}(X_i^{*\top}\beta,\ \sigma^2) \\ T_i^* &\overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,\ \sigma^2) \end{aligned}$$

- Use weighted regression for outcome model
- further transformation of T_i^* can make these distributional assumptions more credible

CBPS for a Continuous Treatment

- Covariate Balancing Generalized Propensity Score (CBGPS)
- Estimate the generalized propensity score such that covariate balance is optimized
- Covariate balancing condition:

$$\begin{aligned}\mathbb{E}\left(w_i T_i^* X_i^*\right) &= \int \left\{ \int \frac{f(T_i^*)}{f(T_i^* \mid X_i^*)} T_i^* dF(T_i^* \mid X_i^*) \right\} X_i^* dF(X_i^*) \\ &= \mathbb{E}(T_i^*)\mathbb{E}(X_i^*) = 0.\end{aligned}$$

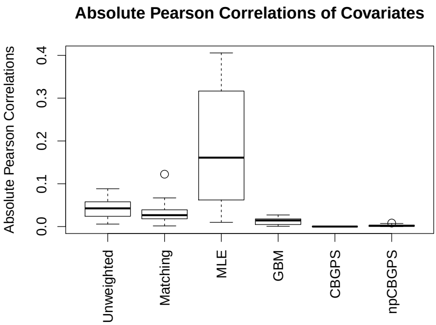
- Combine them with the score condition for σ^2
- Nonparametric CBGPS based on empirical likelihood (npCBGPS)

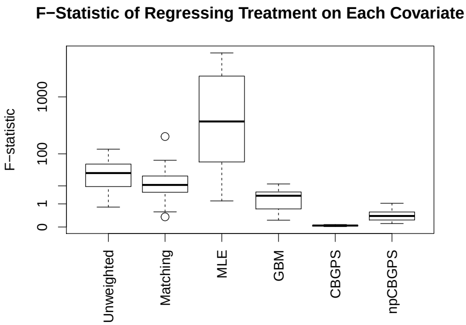
Empirical Application

- Effect of advertisements on campaign contributions
- Urban and Niebler (2014) exploit the fact that media markets cross state boundaries
- Candidates inadvertently advertise in non-competitive states
- Do TV advertisements increase campaign contributions?

- T_i: Number of advertisements aired in each zip code
- ranges from 0 to 22,379 advertisements
- Original analysis $\rightsquigarrow$ dichotomization (over 1000 vs. less than 1000)
- Propensity score matching followed by linear regression with an original treatment variable

Covariate Balance





	Unweighted	MLE	GBM	CBGPS
log(Population)	−0.059	−0.034	0.016	0.000
% Over 65	0.006	−0.162	−0.004	−0.000
log(Income + 1)	−0.021	−0.384	0.014	−0.000
% Hispanic	−0.043	0.053	0.007	0.000
% Black	−0.076	0.295	−0.003	0.000
Population Density	−0.088	0.405	0.016	−0.000
% College Graduates	−0.032	−0.145	0.018	−0.000
Can Commute	0.054	0.161	0.027	−0.000
log(Population)^2	−0.057	−0.049	0.018	0.000
% Over 65^2	0.010	−0.071	−0.001	0.000
log(Income + 1)^2	−0.028	−0.338	0.018	−0.000
% Hispanic^2	−0.013	−0.010	0.006	0.000
% Black^2	−0.057	0.291	−0.007	0.000
Population Density^2	−0.072	0.406	0.003	−0.000
% College Graduates^2	−0.028	−0.079	0.022	0.000

Estimated Effect of Political Advertisements

Method	Estimate	Standard Error	95% CI
Matching (original)	6800	1655	(3556, 10043)
MLE	477	4629	(-345, 17532)
GBM	11176	2555	(6105, 16095)
CBGPS	4935	3865	(-1032, 13989)
npCBGPS	6518	3668	(-415, 13840)

Causal Inference with Longitudinal Data

- Setup:
  - units: $i = 1, 2, \ldots, n$
  - time periods: $j = 1, 2, \ldots, J$
  - fixed J with $n \longrightarrow \infty$
  - time-varying binary treatments: $T_{ij} \in \{0, 1\}$
  - treatment history up to time j: $\overline{T}_{ij} = \{T_{i1}, T_{i2}, \ldots, T_{ij}\}$
  - time-varying confounders: X_{ij}
  - confounder history up to time j: $\overline{X}_{ij} = \{X_{i1}, X_{i2}, \ldots, X_{ij}\}$
  - outcome measured at time J: Y_i
  - potential outcomes: $Y_i(\bar{t}_J)$
- Assumptions:
  1. Sequential ignorability

     $$Y_i(\bar{t}_J) \perp\!\!\!\perp T_{ij} \mid \overline{T}_{i,j-1} = \bar{t}_{j-1}, \overline{X}_{ij} = \bar{x}_j$$

     where $\bar{t}_J = (\bar{t}_{j-1}, t_j, \ldots, t_J)$
  2. Common support

     $$0 < \Pr(T_{ij} = 1 \mid \overline{T}_{i,j-1}, \overline{X}_{ij}) < 1$$

Inverse-Probability-of-Treatment Weighting

- Weighting each observation via the inverse probability of its observed treatment sequence (Robins 1999)
- Inverse-Probability-of-Treatment Weights:

$$w_i = \frac{1}{P(\overline{T}_{iJ} \mid \overline{X}_{iJ})} = \prod_{j=1}^{J} \frac{1}{P(T_{ij} \mid \overline{T}_{i,j-1}, \overline{X}_{ij})}$$

- Stabilized weights:

$$w_i^* = \frac{P(\overline{T}_{iJ})}{P(\overline{T}_{iJ} \mid \overline{X}_{iJ})}$$

Marginal Structural Models (MSMs)

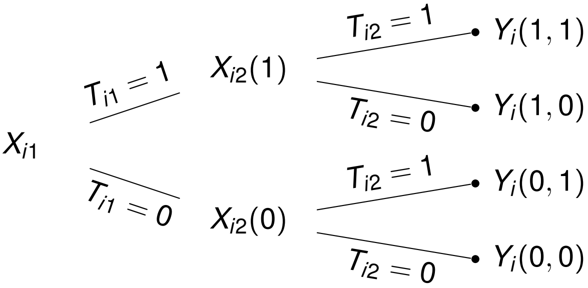
- Consistent estimation of the marginal mean of potential outcome:

$$\frac{1}{n}\sum_{i=1}^{n} \mathbf{1}\{\overline{T}_{iJ} = \overline{t}_J\} w_i Y_i \xrightarrow{p} \mathbb{E}(Y_i(\overline{t}_J))$$

- In practice, researchers fit a weighted regression of Y_i on a function of $\overline{T}_{iJ}$ with regression weight w_i
- Adjusting for $\overline{X}_{iJ}$ leads to post-treatment bias
- MSMs estimate the average effect of any treatment sequence

- **Problem:** MSMs are sensitive to the misspecification of treatment assignment model (typically a series of logistic regressions)
- The effect of misspecification can propagate across time periods
- **Solution:** estimate MSM weights so that covariates are balanced

Two Time Period Case



- time 1 covariates X_{i1}: 3 equality constraints

$$\mathbb{E}(X_{i1}) = \mathbb{E}[\mathbf{1}\{T_{i1} = t_1, T_{i2} = t_2\} w_i X_{i1}]$$

- time 2 covariates X_{i2}: 2 equality constraints

$$\mathbb{E}(X_{i2}(t_1)) = \mathbb{E}[\mathbf{1}\{T_{i1} = t_1, T_{i2} = t_2\} w_i X_{i2}(t_1)]$$

for $t_2 = 0, 1$

Orthogonalization of Covariate Balancing Conditions

	Treatment history: (t_1, t_2)				
Time period	(0,0)	(0,1)	(1,0)	(1,1)	Moment condition
	$+$	$+$	$-$	$-$	$\mathbb{E}\left\{(-1)^{T_{i1}} w_i X_{i1}\right\} = 0$
time 1	$+$	$-$	$+$	$-$	$\mathbb{E}\left\{(-1)^{T_{i2}} w_i X_{i1}\right\} = 0$
	$+$	$-$	$-$	$+$	$\mathbb{E}\left\{(-1)^{T_{i1}+T_{i2}} w_i X_{i1}\right\} = 0$
time 2	$+$	$-$	$+$	$-$	$\mathbb{E}\left\{(-1)^{T_{i2}} w_i X_{i2}\right\} = 0$
	$+$	$-$	$-$	$+$	$\mathbb{E}\left\{(-1)^{T_{i1}+T_{i2}} w_i X_{i2}\right\} = 0$

GMM Estimator (Two Period Case)

- Independence across balancing conditions:

$$\hat{\beta} = \underset{\beta \in \Theta}{\text{argmin}} \ \text{vec}(\mathbf{G})^\top \widehat{\mathbf{W}}^{-1} \text{vec}(\mathbf{G})$$

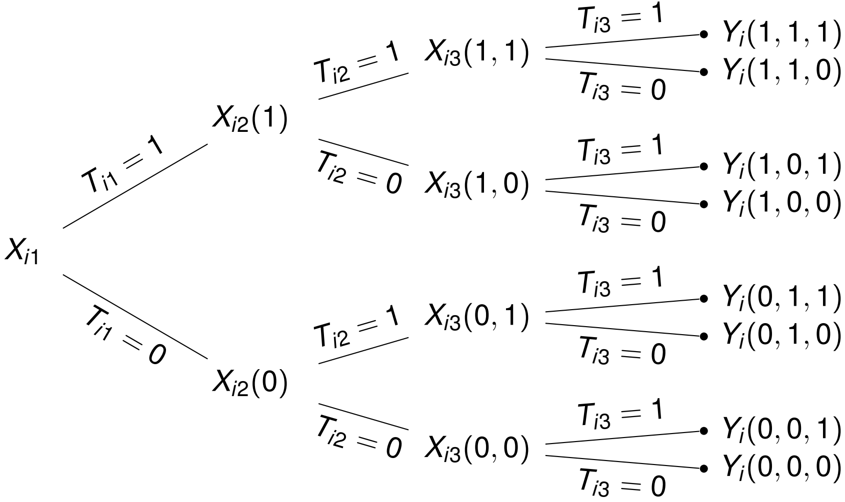
- Sample moment conditions **G**:

$$\frac{1}{n}\sum_{i=1}^{n} \begin{bmatrix} (-1)^{T_{i1}} w_i X_{i1} & (-1)^{T_{i2}} w_i X_{i1} & (-1)^{T_{i1}+T_{i2}} w_i X_{i1} \\ 0 & (-1)^{T_{i2}} w_i X_{i2} & (-1)^{T_{i1}+T_{i2}} w_i X_{i2} \end{bmatrix}$$

- Covariance matrix **W**:

$$\frac{1}{n}\sum_{i=1}^{n} \mathbb{E}\left\{ \begin{bmatrix} 1 & (-1)^{T_{i1}+T_{i2}} & (-1)^{T_{i2}} \\ (-1)^{T_{i1}+T_{i2}} & 1 & (-1)^{T_{i1}} \\ (-1)^{T_{i2}} & (-1)^{T_{i1}} & 1 \end{bmatrix} \otimes w_i^2 \begin{bmatrix} X_{i1}X_{i1}^\top & X_{i1}X_{i2}^\top \\ X_{i2}X_{i1}^\top & X_{i2}X_{i2}^\top \end{bmatrix} \middle| \mathbf{X}_i \right\}$$

Extending Beyond Two Period Case



Generalization of the proposed method to J periods is in the paper

Orthogonalized Covariate Balancing Conditions

Design matrix			Treatment History Hadamard Matrix: (t_1, t_2, t_3)								Time		
			(0,0,0)	(1,0,0)	(0,1,0)	(1,1,0)	(0,0,1)	(1,0,1)	(0,1,1)	(1,1,1)			
T_{i1}	T_{i2}	T_{i3}	h_0	h_1	h_2	h_{12}	h_{13}	h_3	h_{23}	h_{123}	1	2	3
$-$	$-$	$-$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	$+$	✗	✗	✗
$+$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	✓	✗	✗
$-$	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	✓	✓	✗
$+$	$+$	$-$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	✓	✓	✗
$-$	$-$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	✓	✓	✓
$+$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$	$-$	$+$	✓	✓	✓
$-$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$+$	$+$	✓	✓	✓
$+$	$+$	$+$	$+$	$-$	$-$	$+$	$-$	$+$	$+$	$-$	✓	✓	✓

- The mod 2 discrete Fourier transform:

$$\mathbb{E}\{(-1)^{T_{i1}+T_{i3}} w_i X_{ij}\} = 0 \quad (\text{6th row})$$

- Connection to the fractional factorial design
  - "Fractional" = past treatment history
  - "Factorial" = future potential treatments

GMM in the General Case

- The same setup as before:

$$\hat{\beta} = \underset{\beta \in \Theta}{\operatorname{argmin}} \ \mathrm{vec}(\mathbf{G})^\top \widehat{\mathbf{W}}^{-1} \mathrm{vec}(\mathbf{G})$$

where

$$\mathbf{G} = \frac{1}{n} \sum_{i=1}^{n} \left(M_i^\top \otimes w_i X_i \right) \mathbf{R}$$

$$\mathbf{W} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E} \left(M_i M_i^\top \otimes w_i^2 X_i X_i^\top \mid X_i \right)$$

- M_i is the $(2^J - 1)$th row of *model matrix* based on the design matrix in Yates order
- For each time period j, define the *selection matrix* $\mathbf{R}$

$$\mathbf{R} = [\mathbf{R}_1 \ \ldots \mathbf{R}_J] \quad \text{where} \quad \mathbf{R}_j = \begin{bmatrix} \mathbf{0}_{2^{j-1} \times 2^{j-1}} & \mathbf{0}_{2^{j-1} \times (2^J - 2^{j-1})} \\ \mathbf{0}_{(2^J - 2^{j-1}) \times 2^{j-1}} & \mathbf{I}_{2^J - 2^{j-1}} \end{bmatrix}$$

Empirical Illustration: Negative Advertisements

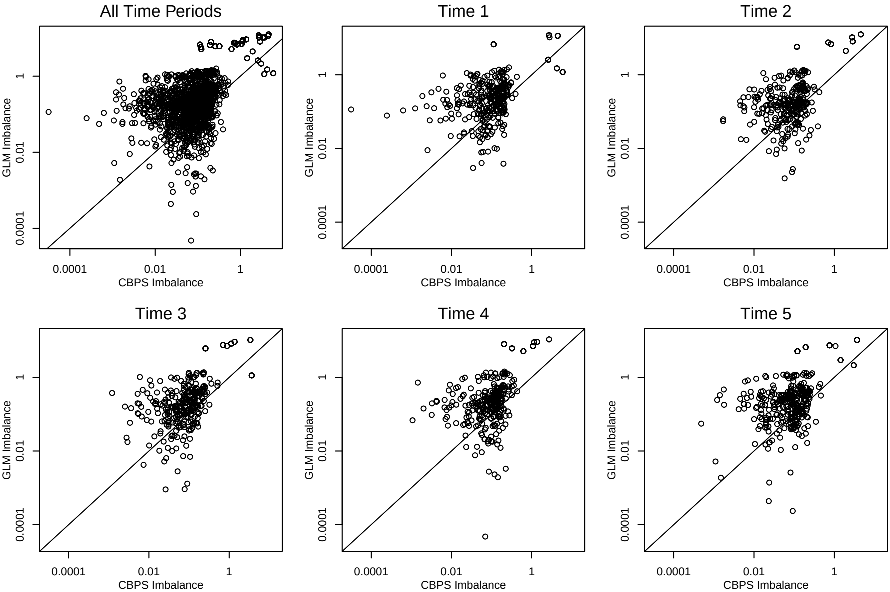
- Electoral impact of negative advertisements (Blackwell, 2013)
- For each of 114 races, 5 weeks leading up to the election

- Outcome: candidates' voteshare
- Treatment: negative ($T_{it} = 1$) or positive ($T_{it} = 0$) campaign

- Time-varying covariates: Democratic share of the polls, proportion of voters undecided, campaign length, and the lagged and twice lagged treatment variables for each week
- Time-invariant covariates: baseline Democratic voteshare, baseline proportion undecided, and indicators for election year, incumbency status, and type of office

- Original study: pooled logistic regression with a linear time trend
- We compare period-by-period GLM with CBPS

Covariate Balance



	GLM	CBPS	CBPS (approx.)	GLM	CBPS	CBPS (approx.)
(Intercept)	55.69*	57.15*	57.94*	55.41*	57.06*	57.73*
	(4.62)	(1.84)	(2.12)	(3.09)	(1.68)	(1.88)
Negative	2.97	5.82	3.15			
(time 1)	(4.55)	(5.30)	(3.76)			
Negative	3.53	2.71	5.02			
(time 2)	(9.71)	(9.26)	(8.55)			
Negative	−2.77	−3.89	−3.63			
(time 3)	(12.57)	(10.94)	(11.46)			
Negative	−8.28	−9.75	−10.39			
(time 4)	(10.29)	(7.79)	(8.79)			
Negative	−1.53	−1.95*	−2.13*			
(time 5)	(0.97)	(0.96)	(0.98)			
Negative				−1.14	−1.35*	−1.51*
(cumulative)				(0.68)	(0.39)	(0.43)
R^2	0.04	0.14	0.13	0.02	0.10	0.10
F statistics	0.95	3.39	3.32	2.84	12.29	12.23

Concluding Remarks

- Covariate balancing propensity score:
  1. optimizes covariate balance
  2. is robust to model misspecification
  3. improves inverse probability weighting methods

- Ongoing work:
  1. Many covariates $\rightsquigarrow$ confounder selection
  2. Generalizing instrumental variable estimates
  3. Spatial causal inference

- Open-source software, `CBPS: R Package for Covariate Balancing Propensity Score`, is available at CRAN

References

1. "Covariate Balancing Propensity Score" *J. of the Royal Statistical Society, Series B (Methodological)*. (2014)

2. "Covariate Balancing Propensity Score for a Continuous Treatment: Application to the Efficacy of Political Advertisements " Working paper available at **http://imai.princeton.edu**

3. "Robust Estimation of Inverse Probability Weights for Marginal Structural Models" *Journal of the American Statistical Association*. (2015).

Send comments and suggestions to kimai@Princeton.Edu