Noncompliance in Randomized Experiments

Kosuke Imai

Harvard University

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Encouragement Design

- Often, for ethical and logistical reasons, we cannot force all experimental units to follow the randomized treatment assignment
  1. some in the treatment group refuse to take the treatment
  2. others in the control group manage to receive the treatment

  $\rightsquigarrow$ noncompliance
- Intention-to-Treat (ITT) analysis:
  - ITT effect can be estimated without bias
  - ITT analysis does not yield the treatment effect
- As-Treated analysis
  - comparison of the treated and untreated subjects
  - no benefit of randomization $\rightsquigarrow$ selection bias
- Can we estimate the treatment effect somehow?
- Encouragement design: randomize the encouragement to receive the treatment rather than the receipt of the treatment itself
  $\rightsquigarrow$ attractive to policy makers

Potential Outcomes Notation

- Randomized encouragement: $Z_i \in \{0, 1\}$
- Potential treatment variables: $(T_i(1), T_i(0)) \in \{0, 1\}$
- Observed treatment receipt indicator: $T_i = T_i(Z_i)$
- Potential outcomes: $Y_i(z, t)$
- Observed outcome: $Y_i = Y_i(Z_i, T_i(Z_i))$
- Can be written as $Y_i(z)$ and $Y_i = Y_i(Z_i)$
- No interference between units for $T_i(z)$ and $Y_i(z)$
- Randomization of encouragement:

$$(Y_i(1), Y_i(0), T_i(1), T_i(0)) \perp\!\!\!\perp Z_i$$

- But, the treatment is NOT random

$$(Y_i(1), Y_i(0)) \not\perp\!\!\!\perp T_i \mid Z_i = z$$

Principal Stratification (Angrist, et al. 1996. *J. Am. Stat. Assoc*)

- Four principal strata (latent types):
  - complier $(T_i(1), T_i(0)) = (1, 0)$,
  - non-complier $\begin{cases} always-taker & (T_i(1), T_i(0)) = (1, 1), \\ never-taker & (T_i(1), T_i(0)) = (0, 0), \\ defier & (T_i(1), T_i(0)) = (0, 1) \end{cases}$

- Observed and principal strata:

	$Z_i = 1$	$Z_i = 0$
$T_i = 1$	Compliers/Always-takers	Defiers/Always-takers
$T_i = 0$	Defiers/Never-takers	Compliers/Never-takers

Instrumental Variables

- Assumptions:
  1. Randomized encouragement as an instrument for the treatment
  2. Monotonicity: No defiers

     $$T_i(1) \geq T_i(0) \quad \text{for all } i.$$

  3. Exclusion restriction: Instrument (encouragement) affects outcome only through treatment

     $$Y_i(1, t) \;=\; Y_i(0, t) \quad \text{for } t = 0, 1$$

     Zero ITT effect for always-takers and never-takers

- ITT effect decomposition:

$$\begin{aligned}
\text{ITT} &= \text{ITT}_c \times \Pr(\text{compliers}) + \underbrace{\text{ITT}_a}_{=0 \text{ by excl. rest.}} \times \Pr(\text{always-takers}) \\
&\quad + \underbrace{\text{ITT}_n}_{=0 \text{ by excl. rest.}} \times \Pr(\text{never-takers}) + \text{ITT}_d \times \underbrace{\Pr(\text{defiers})}_{=0 \text{ by monotonicity}} \\
&= \text{ITT}_c \times \Pr(\text{compliers})
\end{aligned}$$

Identifying the Proportion of Compliers

- Under the monotonicity:

	$Z_i = 1$	$Z_i = 0$
$T_i = 1$	Compliers/Always-takers	~~Defiers~~/Always-takers
$T_i = 0$	~~Defiers~~/Never-takers	Compliers/Never-takers

- Complier proportion equals the ITT effect of encouragement on treatment receipt

$$
\begin{aligned}
& \mathbb{E}(T_i(1) - T_i(0)) \\
= \; & \Pr(T_i = 1 \mid Z_i = 1) - \Pr(T_i = 1 \mid Z_i = 0) \text{ (by randomization)} \\
= \; & \Pr(\text{compliers and always-takers}) - \Pr(\text{always-takers}) \\
= \; & \Pr(\text{compliers})
\end{aligned}
$$

IV Estimand and Interpretation

- Recall: ITT $=$ $\text{ITT}_c \times \Pr(\text{compliers})$
- $\text{ITT}_c =$ ATE for compliers
- IV estimand:

$$
\begin{aligned}
\text{ITT}_c &= \frac{\text{ITT}}{\Pr(\text{compliers})} \\
&= \frac{\mathbb{E}(Y_i \mid Z_i = 1) - \mathbb{E}(Y_i \mid Z_i = 0)}{\mathbb{E}(T_i \mid Z_i = 1) - \mathbb{E}(T_i \mid Z_i = 0)} \\
&= \frac{\text{Cov}(Y_i, Z_i)}{\text{Cov}(T_i, Z_i)}
\end{aligned}
$$

- $\text{ITT}_c =$ Complier Average Treatment Effect (CATE)
- Local Average Treatment Effect (LATE)
- CATE $\neq$ ATE unless ATE for noncompliers equals CATE
- Different encouragement (instrument) yields different compliers

Asymptotic Inference

- Wald estimator: $\widehat{\text{IV}}_{\text{Wald}} = \frac{\widehat{\text{Cov}(Y_i, Z_i)}}{\widehat{\text{Cov}(T_i, Z_i)}} = \frac{\widehat{\text{ITT}}_Y}{\widehat{\text{ITT}}_T}$
- Identical to the two-stage least squares estimator:
  1. Regress T_i on Z_i and obtain fitted values $\widehat{T}_i$
  2. Regress Y_i on $\widehat{T}_i$
- Consistency: $\widehat{\text{IV}}_{\text{Wald}} \xrightarrow{p} \text{CATE} = \text{ITT}_c$
- Asymptotic variance via the Delta method:

$$\mathbb{V}(\widehat{\text{IV}}_{\text{Wald}}) \approx \frac{1}{\text{ITT}_T^4}\Big\{\text{ITT}_T^2\,\mathbb{V}(\widehat{\text{ITT}}_Y) + \text{ITT}_Y^2\,\mathbb{V}(\widehat{\text{ITT}}_T) - 2\,\text{ITT}_Y\,\text{ITT}_T\,\text{Cov}(\widehat{\text{ITT}}_Y, \widehat{\text{ITT}}_T)\Big\}.$$

where

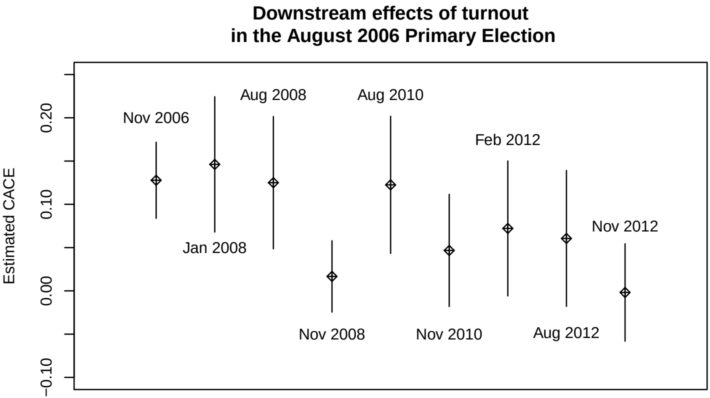
$$\text{Cov}(\widehat{\text{ITT}}_Y, \widehat{\text{ITT}}_T) = \frac{\text{Cov}(Y_i(1), T_i(1))}{n_1} + \frac{\text{Cov}(Y_i(0), T_i(0))}{n_0}$$

Testing Habitual Voting (Coppock and Green. 2016. *Am. J. Political Sci.*)

- Settings (Revisit the Social Pressure Experiment):
  - Randomized encouragement to vote in the 2006 August primary
  - Treatment: turnout in the 2007 November municipal election
  - Outcome: turnout in the 2008 January party primary and subsequent elections
- Assumptions:
  1. Monotonicity: Being contacted by a canvasser would *never* discourage anyone from voting
  2. Exclusion restriction: being contacted by a canvasser in this election has no effect on turnout in the next election other than through turnout in this election
- CATE: Habitual voting for those who would vote if and only if they are contacted by a canvasser in this election

Downstream Effects

- Estimated proportion of principal strata:
  - compliers: est. = 0.083, s.e. = 0.003
  - always-takers: est. = 0.311, s.e. = 0.001
  - never-takers: est. = 0.606, s.e. = 0.003
- CATE:



Violations of IV Assumptions

1. Violation of exclusion restriction:

$$\text{bias} = \text{ITT}_{\text{noncomplier}} \times \frac{\Pr(\text{noncomplier})}{\Pr(\text{complier})}$$

   - Weak encouragement (instruments)
   - Direct effects of encouragement; failure of randomization, alternative causal paths

2. Violation of monotonicity:

$$\text{bias} = (\text{CATE} + \text{ITT}_{\text{defier}}) \times \frac{\Pr(\text{defier})}{\Pr(\text{complier}) - \Pr(\text{defier})}$$

   - Proportion of defiers
   - Heterogeneity of causal effects

Back to the Habitual Voting Example

- Effect of voting in 2006 election on the turnout in the 2008 election: est $= 0.128$, s.e. $= 0.022$
- Potential bias of estimated CATE due to exclusion restriction:

$$\text{ITT}_{noncomplier} \times \frac{1 - 0.083}{0.083} = \text{ITT}_{noncomplier} \times 11.05$$

Summary

- Noncompliance in randomized experiments
- ITT vs. CATE (LATE) $\rightsquigarrow$ additional assumptions are required
  1. randomization of instrument
  2. monotonicity
  3. exclusion restriction
- Traditional instrumental variables $\rightsquigarrow$ ignoring heterogeneity

- Problems of external validity:
  - compliers vs. noncompliers
  - compliers as latent group defined by an instrument